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A SPECTROPHOTOMETRIC STUDY OF CERTAIN CASES OF STRUCTURAL COLOR[1]

By Ernest Merritt

In the past there has been considerable uncertainty and in some instances difference of opinion as to the origin of the brilliant iridescent colors exhibited by the wings of certain butterflies, by the feathers of many birds—for example the tail feathers of the peacock—and by insects like the Brazilian beetle. By their luster and brilliancy and by their general appearance these colors often suggest those produced by metallic reflection from highly absorbing dyes like fuchsin; but the startling changes in color resulting from changes in the angle at which the surface is observed are more suggestive of diffraction colors or of colors due to interference in thin films.

An extended investigation of the subject recently undertaken by Bancroft, Mason and others[2] has led to the conclusion that the colors in question are rarely due to the presence of any colored pigment, but are in the great majority of cases structural colors. While the non-iridescent blues and some greens are often due to scattering produced by small particles or cavities, the most striking cases of color produced without the presence of colored pigment appear to be due to interference in thin films. Although the evidence upon which this conclusion is based is in the main convincing, it is indirect and almost exclusively of a qualitative nature, so that it has seemed to me worth while to

[1] The investigation of which the work here described forms a part has been supported by a grant from the Heckscher Foundation, established at Cornell University by August Heckscher.

[2] Structural Colors in Feathers, C. W. Mason. Journ. Phys. Chem. *27*, pp. 201-251, and 401-447; 1923.

Blue Feathers. W. D. Bancroft, E. M. Chamot, E. Merritt, and C. W. Mason. The Auk. *40*, pp. 275-300; 1923.

check the conclusions, at least in a few cases, by spectrophotometric measurements.

Three cases of structural color have thus far been studied, viz., the neck feathers of a pigeon, the wing of a butterfly (*Morpho menelaus*) and polished steel that had been colored by tempering. In the first case the color might be described as a rich blue-green whose composition changed with the angle of observation. The butterfly wing, a light blue green, was brilliantly iridescent. Several specimens of tempered steel were used, ranging from a pale straw color to a bright blue. Light from a gas filled tungsten lamp, diffused by ground glass, was reflected into one slit of a Lummer-Brodhun spectrophotometer by the specimen and was compared throughout the spectrum with the diffused light from a similar lamp illuminating the other slit. Measurements were made at a number of different angles of reflection.

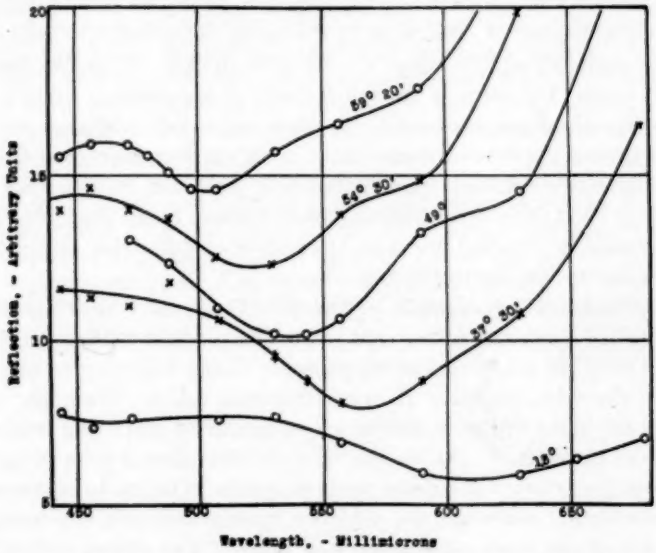

FIG. 1. *Spectral distribution of the light reflected at different angles of incidence from the neck feather of a pigeon.*

The results of such measurements in the case of the pigeon feather are shown in Fig. 1. The fact that the absolute intensity is widely different for different angles of incidence is not significant, since in each case the slit width and the intensity of the light were adjusted more or less arbitrarily. Each curve shows an absorption band—or region of

relatively small reflection—whose position in the spectrum shifts progressively with changing angle of incidence in such a direction as to agree at least qualitatively with the view that the color is due to interference in a thin film. If it is assumed that such a film is present, that its thickness is e, and that its refractive index is n, the usual theory shows that we should have

$$2ne \cos r = k\lambda_0$$

where λ_0 is the wave length of maximum destructive interference, i the angle of incidence and k an integer depending upon the order of interference. The form of the curves of spectral distribution makes it clear that in the case here studied k must be put equal to unity. If, for example, k were 2 we should expect destructive interference not only at λ_0 but also at $\lambda = 2\lambda_0$. Although the appearance of the curves suggests that some secondary interference is superposed upon the main effect there is nothing to indicate the presence of a band in the infrared.

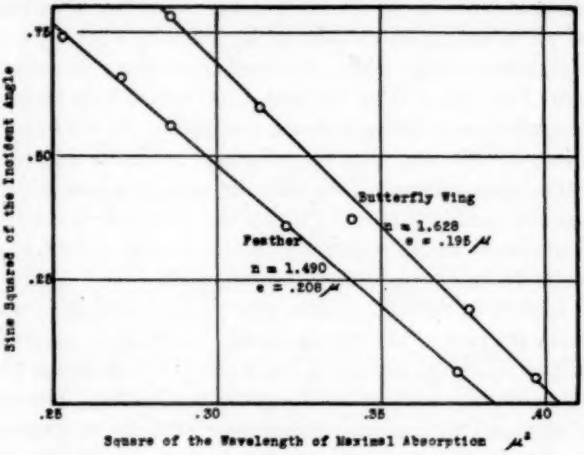

FIG. 2.

The expression for λ_0 may be put into the form

$$\frac{k^2}{4e^2}\lambda_0^2 = n^2 - \sin^2 i$$

If the dispersion of the medium is small so that n may be treated as a constant we should therefore obtain a straight line when λ^2 is plotted against $\sin^2 i$. The result of plotting the observational data in this way is shown in Fig. 2. Although the rather flat minima of the intensity curves make a very accurate determination of λ_0 impossible, yet the

points plotted in Fig. 2 fall nearly enough on a straight line to offer a satisfactory confirmation of the thin film theory. n and e can be computed from the slant and intercept of the line. Such computation shows that the thickness of the film must be 0.208μ and the index of refraction of the material 1.490.

In the case of the butterfly wing the color of the reflected light was found to depend not only on the angle of incidence but also on the position of the plane of incidence with reference to the wing structure. It has long been known that butterfly wings are covered with minute scales, and that these often show fine striations. These striations, in the case of the wing studied, appeared under the microscope very much like the lines of a diffraction grating. The iridescent colors of butterfly wings have in fact often been explained as diffraction colors, although it has been pointed out by Bancroft that grating diffraction could be an important factor only in the case of illumination by a concentrated source of light. Dr. W. T. M. Forbes has called my attention to the fact that when butterflies are seen in the shade of a forest, so as to be brightly illuminated only when they pass through an occasional beam of sunlight, they often show brilliant colors which bear no relation to their color when seen under ordinary conditions. It would seem from observations of this kind that the striations are quite capable of producing diffraction effects. To avoid the complications which might arise from the combined effect of diffraction and thin film interference the measurements whose results are plotted in Fig. 2 were made with the plane of incidence parallel to the striations.

It will be noticed that the results agree with the thin film theory fully as well as in the case of the pigeon feather. Such close agreement was not anticipated, for the surface of the scales is by no means plane but rather resembles the back of an elongated turtle shell. The scales are however arranged with great regularity so that the average angle of incidence is probably nearly the same for all scales. Similar measurements with wings from different species of butterfly and for different planes of incidence would undoubtedly lead to interesting results. In the case studied the observations show that the color is accounted for by a film whose thickness is 0.195μ and whose index of refraction is 1.628.

The measurements of the light reflected from tempered steel were made in order to obtain data, in a case where the cause of the color may be looked upon as definitely known, for comparison with the data obtained in the two cases where the cause is not so well established. The

temper colors of metals, of which the blue color of polished and tempered steel is perhaps the commonest example, have generally been explained ever since the time of Newton as the result of interference in a thin film; and although this explanation has been called in question by Mallock,[3] such convincing evidence has been advanced by Mason[4] in its support that little doubt of the correctness of the usual explanation can remain.

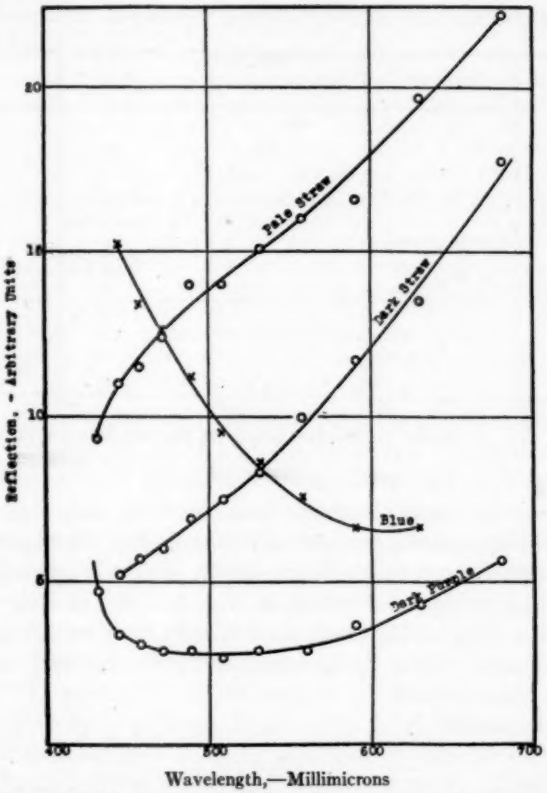

FIG. 3. *Spectral distribution of the light reflected at nearly normal incidence from different specimens of tempered steel.*

The spectral distribution in the light reflected at nearly normal incidence is shown in Fig. 3 for four specimens of steel colored by heating. In the two straw colored specimens the film is so thin that no

[3] Proc. Roy. Soc. 94A, p. 561, 1918.
[4] Jr. Phys. Chem. *28*, pp. 1233-1244; 1924.

interference band appears in the visible range. In the purple and blue specimens minima could be located, although with considerable difficulty, and the lines obtained by plotting $\sin^2 i$ against λ_0^2 are shown in Fig. 4. I need hardly say that the results are subject to large uncertainty and that the value obtained for the index of refraction, which is much smaller than that usually ascribed to iron oxide, cannot be looked upon as very accurate in spite of the close agreement between the values obtained for the two different specimens. It is possible that

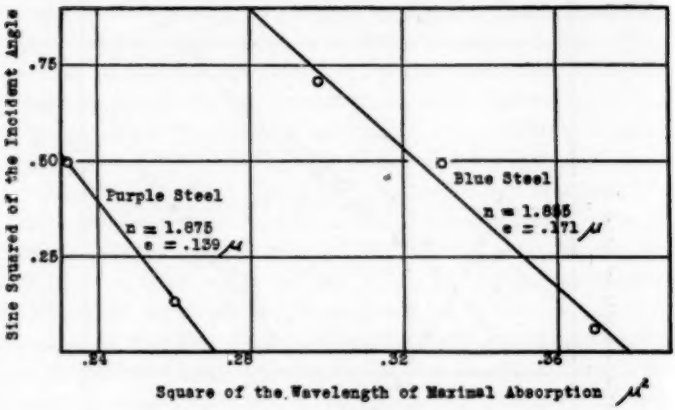

FIG. 4.

the failure of the points plotted for blue steel to lie on a straight line is due to dispersion in the material of the film. But the number of observations is too small to give much weight to such a conclusion. It is probable also, as suggested by Mason, that the oxide film has sufficient body color to appreciably affect the interference colors. A systematic study of temper colors by spectrophotometric methods would undoubtedly be of interest.

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Valence and Arrangement of orbital Electrons.—Bedreag contends that if an atom possesses two or more different valences, each corresponds to a distinct repartition of its orbital electrons in the three or four outer levels. This is defended by citing the familiar observations on the variation of paramagnetic susceptibility with valence, and those on the variation of the x-ray absorption-edges with valence.—[P. Bedreag; C. R. *180*, pp. 653-655; 1925.]

KARL K. DARROW

SPECTRAL ENERGY CHARACTERISTICS OF THE CONSTRICTED MERCURY VAPOR LAMP—AN EXTREMELY CONCENTRATED SOURCE OF ULTRAVIOLET ILLUMINATION

By George Shannon Forbes and George R. Harrison*

ABSTRACT

After having determined in a previous investigation the spectral energy characteristics of a high intensity quartz mercury arc of 8 mm bore for various conditions of operation, the writers here describe a similar set of experiments with an arc constricted to a tube of 2 mm internal diameter and 38 mm long, thus doing away with the outer absorbing layer of relatively cool mercury vapor. Currents of from one-third to five amperes were used, air cooling being necessary between one and two amperes, and water cooling above this point, except at high pressures, when air cooling was necessary even at low currents. Voltage gradients of from seven volts per cm to 58 volts per cm were used. The pressure in the arc, which was of special design, was measured with a mercury manometer, and varied from 29 mm to 3920 mm. The energy in each of the more important groups of lines was measured with a thermopile, and it was found that this type of arc was of very high efficiency, and of tremendous intensity and convenience for slit illumination. Curves are given showing the spectral energy variation with varying current and voltage, and it is shown that approximately the best combination of intensity, efficiency, convenience, and life is obtained at atmospheric pressure, one ampere current, and 110 volts with suitable series resistance. No decrease in efficiency with increasing voltage was observed at pressures up to five atmospheres. Where extreme intensities are desired, currents up to five amperes may be used, but the life of the lamp is shortened.

In a previous paper[1] we have recorded a quantitative study of the effects of variable current, voltage, pressure, and ventilation on fifteen important maxima in the spectrum of a quartz mercury lamp of 8 mm internal diameter. The highest efficiencies attained were observed at two atmospheres pressure, with high voltage gradients, small currents, and a minimum of cooling. Küch and Retschinsky,[2] in 1906, recorded measurements at vapor pressures of three atmospheres in a thick-walled quartz lamp 10 mm in internal diameter, and produced high intensities, but did not separate the effects of the several variables involved. Requiring for photochemical and photographic research a mercury arc capable of furnishing, to a slit, illumination stronger than is possible with lamps hitherto constructed, we were led to investigate in detail the possibilities latent in various extreme conditions of operation.

As is well known, the mercury arc operating at high efficiencies consists of an exceedingly brilliant "thread" surrounded by comparatively

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[1] Harrison and Forbes, This Journal, *10*, p. 1; 1925.

[2] Küch and Retschinsky, Ann. der Phys., *20*, p. 563; 1906.

dark vapor, which absorbs much of its light. We desired to eliminate this absorbing shell, and to study the radiation of the thread alone up to high pressures and high current densities. The lamp shown in Fig. 1 easily withstood pressures up to five atmospheres, and very great energy concentrations. Also, it adjusted itself readily to a wide range of electrical conditions.

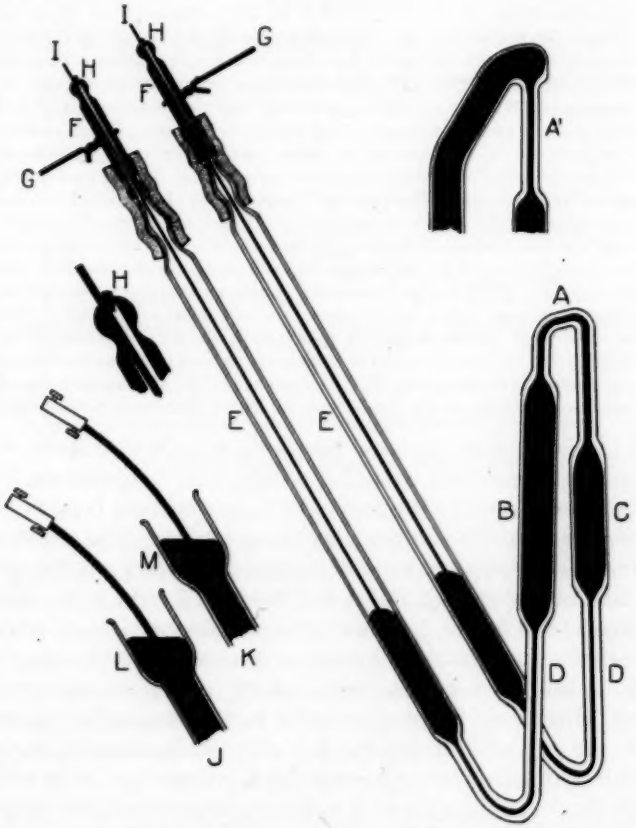

FIG. 1. *Diagram of the constricted arc arranged for pressures up to five atmospheres and currents from 0.2 to 2.0 amperes. The inserts show, at A', the modified form of the constricted portion arranged to have water cool the hot column so that currents up to 5.0 amperes could be used, and below the enlarged ends arranged for work at atmospheric pressure.*

In the first modification of the lamp, the luminous column was confined in a fused quartz tube A, 63 mm in length, 2.0 mm in bore, and 7 mm in external diameter. The upper extremity of the constriction

was made the negative pole so that the upper level would not fall through transfer of mercury by the current. Tubes of 5 mm internal diameter, *B* and *C*, joined its ends to capillaries *DD* about half a millimeter in bore and 11 cm long. These effectively restrained oscillations in the arc, and could carry five amperes without undue heating. The capillaries were bent through 135°, and joined to parallel tubes EE, 8 mm in bore and 20 cm long. These were cemented and wired to a grooved strip of wood between them (not shown) to which a clamp could be safely applied. By supporting the lamp so that *BAC* sloped downwards while *EF* sloped upwards, the latter could within a few minutes be thoroughly boiled out, through the capillaries and beyond, by direct application of a Bunsen flame. No pump or special attachments were needed. Obviously lamps of any size could be boiled out in this manner if similarly constructed.

The open ends of the lamp led to short steel tubes *FF*, soldered to the leads *GG* of a 550 volt dynamo run by an induction motor. Suitable inductance, rheostats, a model 45 standard Weston ammeter and a Model 1 Weston voltmeter completed the electrical equipment.

To attain high pressures it was necessary to add certain attachments unnecessary for operation at atmospheric pressure, for which the lamp was especially suited. From inside of the steel tubes and riveted to them at *H* projected polished steel wires *II*, dipping into the mercury. The steel tubes were suitably connected to vacuum, to a tire pump provided with a valve, and to a pyrex manometer whose open arm was four meters in length. All tubes terminated in nipples, and all connections were made with extra heavy pressure hose of cotton fabric and rubber, greased and wired. The stopcocks were clamped to avoid blowing out. No leakage whatever now occurred, even with four and a half atmospheres excess of internal pressure.

As will be shown below, this lamp, at high pressures, proved much more effective as an illuminant for a slit than arcs of greater cross section run under typical conditions. The pressure equipment, however, was cumbersome and potentially dangerous. Also it was clear that a lamp capable of operation on a 110 volt circuit would be much more widely useful. Therefore, after the desired measurements had been completed, alterations were made as shown in the inserts of Fig. 1. The constriction *A'* was now only 38 mm long. The ends of the tubes, *JK*, were now open to the atmosphere and the enlargements *LM* retarded changes in level due to transfer of mercury. By placing the ends of a *long* U-shaped trough under the end of *L* and above that of

M, the mercury transferred to the upper (negative) electrode will ultimately trickle down the trough, without danger of short circuits, to the positive side, and make the operation of the lamp automatic. The modified form required at least three amperes for its operation, a point discussed at greater length below.

Vincent and Biggs[3] have recently described a capillary arc with a luminous column 0.3 mm in diameter, open to the atmosphere. This operated on 100 to 150 volts, and drew from 0.10 to 0.16 ampere. Also, Harrington[4] has worked out a capillary arc of pyrex glass, with sealed-in tungsten leads, operating under low pressures at 10 volts/cm. The low current capacity of the former type of arc, and the low voltage gradient of the latter imposed serious limitations upon their radiation. The arcs described in the present paper were designed to overcome such limitations.

The lamp could be started at almost any pressure, if sufficient current was used, by warming the capillary with a Bunsen flame. Cooling, when desirable, was effected by a bent perforated copper tube carrying air under pressure, or by an electric fan, or both. Above two amperes a stream of water flowing over the whole constricted portion was necessary, to provide sufficient cooling. Owing to the thickness of walls of the constriction, no condensation of mercury vapor occurred. The lamp always reached a steady state in a minute or less after a change in conditions, and with a minimum of the manipulation described in the previous paper. The wider and thinner-walled tubes *B* and *C* joined to the constriction *A*, radiated heat so much more rapidly than *A*, especially under drastic cooling, that any extensions of the arc into them tended to collapse. No artificial cooling of any sort was necessary, at atmospheric pressure, when the current was less than one ampere. To measure the arc length exactly, an image, magnified six times, was projected upon a sheet of coordinate paper, and the correct value calculated from this for each set of conditions. This arrangement was much more convenient than the cathetometer used in the previous work, and any change in arc length could be immediately noted and recorded. All potential gradients were reduced to volts per centimeter after subtraction of 12 volts for the sum of the terminal voltage drops. This correction should be regarded as a minimum value, as the resistance of the luminous column was much greater than in arcs of greater diameter.

[3] Vincent and Biggs, Jour. Sci. Inst., *1*, p. 242; 1924.
[4] Harrington, This Journal, *7*, p. 689; 1923.

The spectro-radiometer, galvanometer and accessories were identical with those described in the previous paper. The method of measurement was also identical, except that correction for background was made by direct subtraction of minimum readings on either side of the maxima observed. In some cases the energy received by the thermopile was so great that resistances up to ten times the combined resistance of thermopile and galvanometer had to be introduced to keep the deflections below 200 cm. In such cases data for overlapping portions of curves were obtained, and the proper reduction factors computed from average ratios of ordinates taken from the smoothed curves. As in the previous paper, we were interested only in the varying relative intensities of single lines.

In Table 1 are listed representative cases to illustrate the general effects of changing the variables. The first column gives total pressures inside the arc, the second voltage gradients calculated on the basis of a 12-volt end correction, the third current strength, and the last the number of microwatts per square millimeter received at the exit slit for the maximum which comprises the group of lines 3650-63 A. The galvanometer-thermopile combination was calibrated in terms of two standard lamps obtained from the Bureau of Standards, and it was found that 1 mm scale deflection corresponded to 0.72 ergs/mm^2 per sec, the deflections being proportional to light intensity within the limit of observation.

TABLE 1

Pressure mm	Voltage Gradient volts/cm	Current amps	Energy in Lines 3650-63 in watts/mm^2 $\times 10^{-4}$
88	11.7	.33	7.00
654	28.4	.33	39.0
3920	57.6	.33	268.
29	7.4	1.00	15.0
208	13.8	1.00	43.0
664	21.4	1.00	137.
1398	28.2	1.00	392.
752	21.7	3.02	940.
752	27.8	4.00	2090.
752	29.5	5.00	2600.

The extreme values of each variable are not included in the table, but a fifteen-fold variation in both voltage and current was made, though not simultaneously, which resulted in a one hundred and

sixty-fold variation in pressure, and a four hundred-fold variation in light intensity for the 3660 A maximum, while the other maxima varied in certain cases more, and in others less.

In Fig. 2 are given curves showing the relation between galvanometer deflections and voltage gradients in the arc for six of the most important maxima, these being the same groups of lines as were listed in the previous paper. These curves were taken for a current of approximately one ampere. Data for similar curves were taken at .33 amps. (12 to 58 volts/cm), shown in Fig. 3. The data for six pairs of runs under

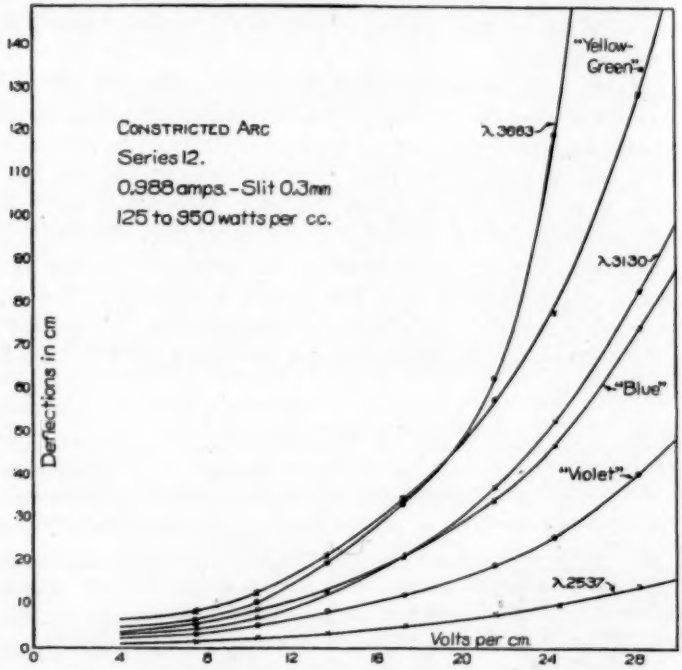

FIG. 2. *Curves show the relation between voltage gradient and galvanometer deflection, at constant current, for six of the more important maxima in the light from the constricted arc.*

nearly the same conditions have been averaged. The pressures in the latter case ranged from one-tenth to five atmospheres. The curves shown are quite similar to those previously published for the arc of large cross-section, the chief difference being in the voltage gradients at which the curves begin to ascend steeply. The first arc was about 50 sq. mm in cross-section, while the present one was slightly over three,

or about one-sixteenth as great. The curves for the larger arc show a sharper bend and a straighter upper portion of the curve, while those for the constricted arc show more uniform curvature. The bends in

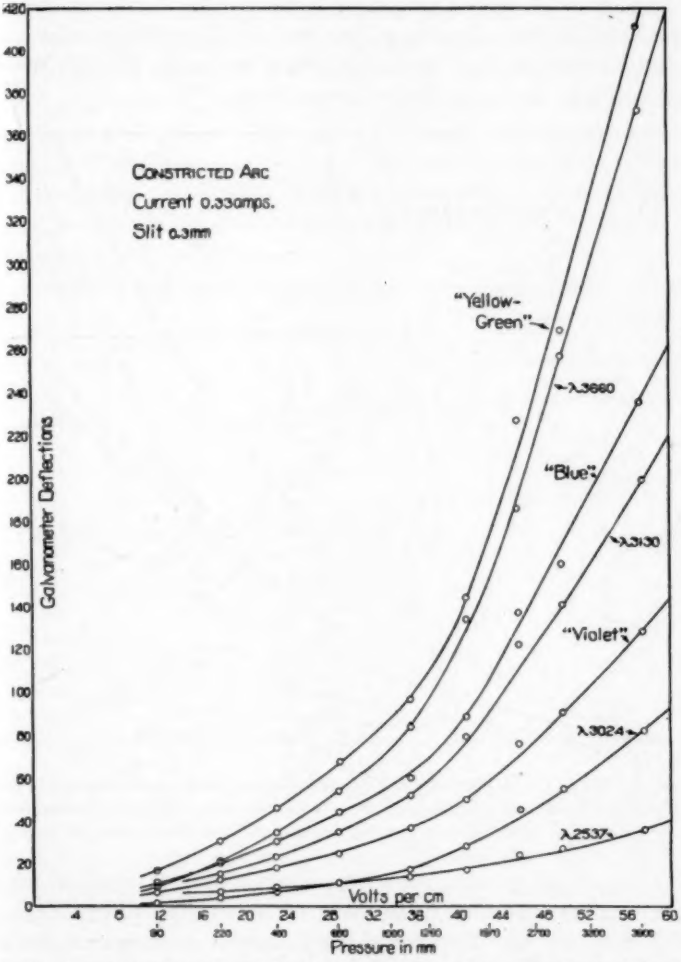

FIG. 3. *Curves similar to those of Fig. 2, but with one-third the current density and twice the voltage range. Pressures are marked below the voltage scale. Galvanometer deflections above 180 cm were measured with a resistance in series with the thermopile and were then reduced to the same scale as those without resistance.*

the curves for the former occur at about six volts/cm, while the corresponding region for the latter seems to be at about twenty volts/cm.

One object of going up to higher pressures was to see if these voltage-deflection curves did not bend over above a certain point, but up to five atmospheres no trace of any such decrease in efficiency was found.

The curves all show a very rapid increase of the maximum at 3663 A. with both current and voltage, this line, alway one of the most intense, greatly out-distancing the yellow-green maximum at high pressures and voltages, above a certain current density.

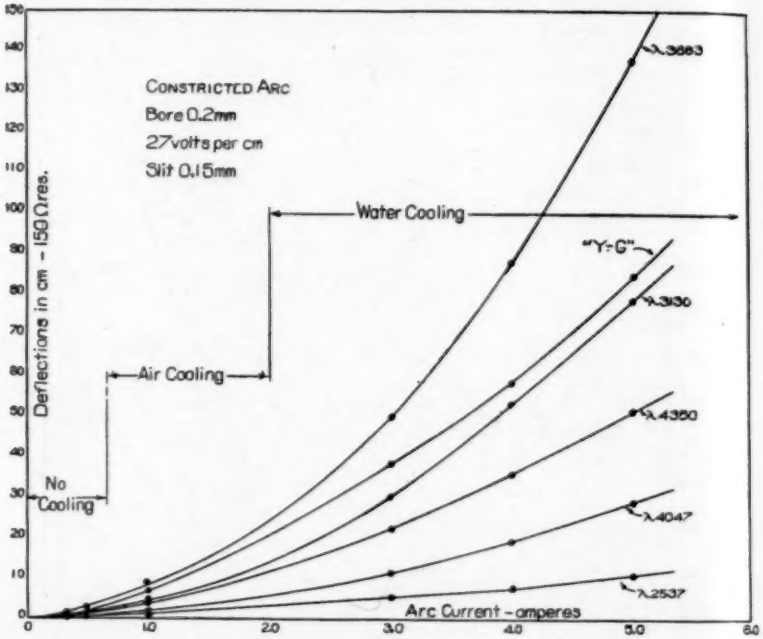

FIG. 4. *Curves showing the relation of galvanometer deflection to current density, for constant voltage gradient. The type of cooling used in each region is marked. Note that readings have been greatly reduced by a series resistance, and that the slit is narrower than in previous cases.*

In Fig. 4 are plotted deflections against current densities for the same maxima, all values being taken for a voltage gradient of twenty-seven volts per cm. Here again the yellow-green maximum, the blue, and the violet go up together, while λ3663 and λ3130 go up together but more rapidly. This is to be expected, since the last two sets contain members of the diffuse triplet series, while the first three largely contain sharp series lines. The different methods of artificially cooling the arc are marked on the regions where they were respectively used.

The results indicate the advantage of the constricted arc in decreasing absorption of all lines, but especially of 2537 A., which is of especial importance as it is the shortest wave-length line in the mercury arc of sufficient intensity for general use in photochemical work. It appears that in the constricted arc, line 2537 can be kept at constant intensity by tripling the current from 0.33 to 1.00 amp., and cutting the voltage from 62 to 32 volts/cm, which is of great importance, since this reduces the pressure from about 4000 mm to 1500 mm. In fact, the greatest advantage of this type of arc seems to be in its high internal resistance, this giving higher efficiencies at normal pressures, so that increase in current becomes more nearly equivalent to increase in voltage in the production of higher intensities.

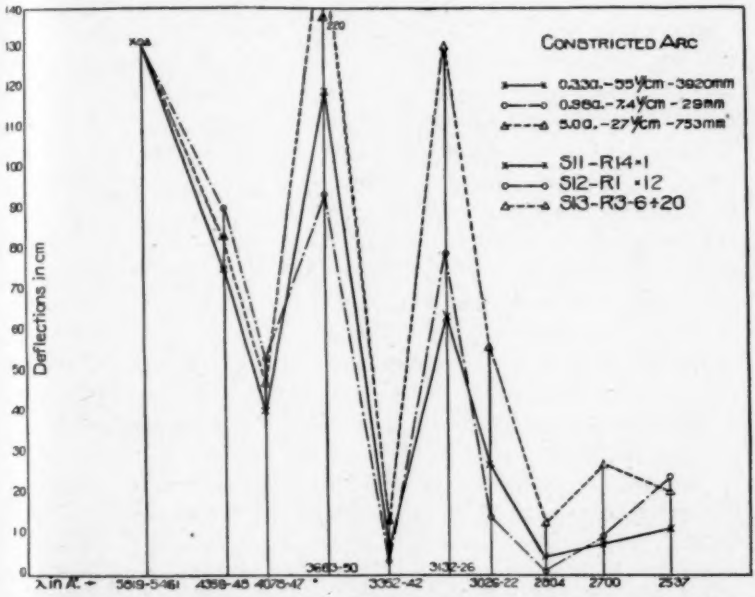

FIG. 5. *Chart showing the variation in intensity distribution for the various maxima observed under different conditions. The solid line is for low current and high voltage gradient, the dash-dot line for moderate current and low voltage, and the dash line for high current and moderate voltage gradient. The yellow-green maximum has been reduced to the same value in each by multiplying by a factor given in the chart. The maxima are directly connected, no minima being marked. The same slit width was used in all three cases.*

In Fig. 5 are shown the changes in energy distribution found for certain extreme cases. The deflections for the yellow-green maximum were reduced to the same value by multiplying deflections for .98 amp

and 7.4 volts/cm by twelve, and dividing those for 5 amps and 27 volts/cm by twenty, while those for 0.33 amps and 55 volts/cm remained the same. The resulting deflections for each maximum so reduced are connected by straight lines, in order to make the relative changes in intensity apparent. The tremendous relative increase in λ3663 at high currents is very striking, while all the ultraviolet maxima except λ2537 show higher relative values at high currents.

The water-cooled constricted arc with a luminous column 38×2 mm, operating under atmospheric pressure, offers many striking advantages. Its initial cost is low, and it is easily repaired if broken. It is portable if the mercury is poured out or if the ends are stoppered. If the vacuum fails or the mercury becomes foul, the lamp can be refilled and boiled out within ten minutes. There are no sealed-in leads to fail, and no possibility of damage through reversing the polarity. There is no tungsten electrode to spatter, the chief reason for loss in the ultraviolet in arcs using this metal. If for any other reason the ultraviolet falls off, treatment with hydrofluoric acid and burning in an oxygen-gas flame restores it; in extreme cases the constricted portion is readily replaced. The lamp reaches a stationary state at least within a minute after lighting. It can be effectively operated, without any cooling, at low power inputs, and the arrangement for water cooling is extremely simple. The absorption, especially of λ2537, is much smaller than in arcs of larger cross-section. The most important advantage, however, is the enormous intensity of slit illumination resulting from the use of high current densities, far exceeding any hitherto attained in mercury vapor lamps. The great loss in efficiency through water cooling observed in arcs of larger cross-section and thinner walls[5] is not observed in the arc here discussed.

As might be expected, something has to be sacrificed to attain the highest intensities, and this sacrifice is in the life of the lamp. Preliminary measurements indicated some decline in radiation over several hours' use, in the extreme ultraviolet, at 3 amperes and 27 volts/cm. When, however, 5 amperes passed under 27 volts/cm, the radiation fell off rapidly. A part, at least, of the decrease was due to carbonization, upon the outside of the capillary, of organic matter. This deposit was readily removed with hydrofluoric acid. We are investigating these matters in greater detail. The fourteen-fold increase in radiation at 5 amperes as compared with 1 ampere might justify rapid deteroriation in certain cases such as study of the reciprocity law, especially in view of

[5] Fabry and Buisson, Compt. Rend., *153*, p. 93; 1911.

the ease of renewal. In a paper intended primarily for chemists we are publishing further data on the changes in radiation of the water-cooled constricted arc, at various wave-lengths, over longer periods of time, and comparing its performance with that of the arc described in the previous paper and with that of a brand-new commercial mercury vapor lamp.

In conclusion, we express our indebtedness to the Rumford Fund of the American Academy of Arts and Sciences for a grant covering the cost of the special lamp used.

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JEFFERSON PHYSICAL LABORATORY (G. R. H.)
HARVARD UNIVERSITY,
FEBRUARY 11, 1925

John A. Brashear: the autobiography of a man who loved the stars. Houghton Mifflin Company, 1925. $4.

The life of John A. Brashear was extraordinary. He experienced a childhood, youth, and early manhood of the poverty-stricken, limited, and toilsome sort which is supposed to be beneficial to men ambitious for success in a lucrative trade or profession, but is rarely anything but an insurmountable hindrance to a man fascinated by pure science. His childhood, spent in a village of the Monongahela valley in the epoch before the Civil War, terminated with his schooling in 1855, when he was fifteen years of age. He then studied bookkeeping in a "mercantile college," worked in a grocery and in an auction store, found satisfaction for the first time in becoming an apprentice to the pattern-making trade, and built engines in Louisville. From 1861 to 1881 he was working in Pittsburgh mills, generally as a millwright—a mechanic who in that day "had one of the hardest and most trying positions to fill in a rolling mill" under conditions compared with which the hot outdoors of Pittsburgh Augusts "seemed like a paradise." Once he remarks, speaking of his life in 1872, that he got up at half-past five to go to work, and would not get home until about six o'clock, "often much later if there had been any kind of breakdown in the mill." It is not apparent that this schedule was ever relaxed until, in 1881 at the age of forty, he broke away to enter upon the life-work on which he had set his heart.

There was something strange and wonderful in the man's personality, which after a day of such toil gave him the incentive and the energy to spend half the night and more in the interminable labor of grinding lenses and mirrors, that he might view the planets and the stars with an instrument of his own. Beginning without instruction or counsel to make an object-glass for a refractor, he eventually took the result of years of toil to Langley at the Allegheny Observatory. Langley inspected and praised the lens, and gave Brashear information which resulted in his undertaking to construct a twelve-inch reflector instead

of a refractor. The passage in which Brashear describes the accident which befell his first mirror, at the very last moment before completion, is pathetic. The second mirror was successfully completed.

Overloaded with odd jobs which he had undertaken for Langley, and others brought to him by an advertisement which he had inserted in the *Scientific American*, offering to make "specula, diagonals and eyepieces for amateurs," Brashear suffered a physical breakdown in 1881, and made the great decision to abandon his work in the mill and devote himself altogether to the making of instruments for astronomers and physicists. Even with his industry, devotion, and modest scale of living, he could not have kept the enterprise afloat but for the assistance of William Thaw, who made him generous subsidies (and incidentally wrote him some sagacious business advice which is entertaining reading). He made instruments of many kinds; one infers that he was proudest of the specula for diffraction-gratings which he made for Rowland. On one occasion Rowland, having obtained a test plane from Munich, informed Brashear that all of his gratings deviated from perfect flatness by the same amount. Brashear made the journey to Baltimore, and found Rowland fixed in that belief. Another physicist persuaded Rowland to test the test plane. The fault was in it.

The remainder of the book is chiefly an account of Brashear's travels, with a valuable interpolated chapter about Langley and another about the organization of the Carnegie Institute.

As a history of the events of the first and formative part of Brashear's life, the autobiography is good. The style is of an artless simplicity, with stereotyped metaphors and turns of phrase which are sometimes amusing and generally engaging. The only regrettable feature is that, whether through lack of interest or lack of skill in self-analysis, Brashear did not enrich his history with a better account of the development of that emotional and intellectual attitude which was evidently his distinction. One forms a picture of a gentle, kindly, child-loving, friend-loving, incredibly laborious and incredibly modest man; but these cannot have been the qualities that made him alternate days in a mill with nights at a telescope, and piled up for him a series of honors of which the list fills four supplementary pages. A few too-brief passages suggest these dominant qualities—the allusion to the "entrancing beauty" of Saturn as he first saw it through a small telescope, at the age of nine; the mention that he studied and composed music during his twenties; the reference to the (unspecified) books that he bought and read during the nights or on the cars; the praise of the craftsmanship displayed by the Nikko temples and the Kamakura statue. What was it that caused the child of Brownsville, the millwright of the South Side of Pittsburgh, to write in later years of the double star Albireo "these two stars are pictures of beauty one can never forget—two brilliant diamonds, orange and blue, shining with their transcendent luster in that far-off setting in yonder blue sky?"

KARL K. DARROW

VISIBILITY OF RADIANT ENERGY EQUATION

By J. W. T. Walsh

In a recent number of this Journal,[1] Tyndall and Gibson have given a four-term expression of the Goldhammer form which represents their recommended values of the visibility function to a considerable degree of accuracy.

As these values have now been provisionally adopted by the International Commission on Illumination it may be of interest to record a similar expression, with slightly different values of the constants, which has been found to give a somewhat closer degree of approximation to

Table 1

	K_λ				K_λ		
λ ($m\mu$)	Adopted	T. & G.	W.	λ ($m\mu$)	Adopted	T. & G.	W.
430	0.0116	0.0008	0.0096	580	0.870	0.863	0.863
440	0.023	0.015	0.021	590	0.757	0.752	0.752
450	0.038	0.033*	0.037	600	0.631	0.626	0.629
460	0.060	0.058	0.061	610	0.503	0.498	0.503
470	0.091	0.090	0.092	620	0.381	0.376	0.378
480	0.139	0.138	0.136	630	0.265	0.268	0.265
490	0.208	0.215	0.207	640	0.175	0.181	0.168
500	0.323	0.337	0.322	650	0.107	0.116	0.103
510	0.503	0.509	0.501	660	0.061	0.071	0.058
520	0.710	0.704	0.707	670	0.032	0.041	0.032
530	0.862	0.851	0.855	680	0.017	0.024	0.016
540	0.954	0.943	0.943	690	0.0082	0.0120	0.0084
550	0.995	0.992	0.994	700	0.0041	0.0067	0.0045
560	0.995	0.985	0.997	710	0.0021	0.0037	0.0024
570	0.952	0.949	0.950	720	0.00105	0.0020	0.0012

* Given in Tyndall & Gibson's paper as 0.030.

the recommended values. This expression was originally calculated to fit the values adopted by the I.E.S. before Gibson and Tyndall's work and it has been modified to suit the differences introduced into these values as a result of this work.

[1] J.O.S.A. & R.S.I., 9, p. 403; Oct. 1924.

The expression is

$$K_\lambda = 0.995\ (R_1 e^{1-R_1})^{220} + 0.105\ (R_2 e^{1-R_2})^{700} + 0.096\ (R_3 e^{1-R_3})^{2000} + 0.055\ (R_4 e^{1-R_4})^{350}$$

where $R_1 = 556/\lambda$, $R_2 = 610/\lambda$, $R_3 = 523/\lambda$ and $R_4 = 475/\lambda$.

In the accompanying table are given the values computed (a) from Gibson and Tyndall's expression and (b) from the above expression.

It will be noticed that the wave length of maximum visibility is taken as 556 mμ instead of 555 mμ.

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Electromagnetic Analysis of Hydrogen Canal-rays.—The canal rays are deflected by electric fields, or magnetic fields, or the two fields crossed, and received upon a phosphorescent screen or upon a movable thermocouple, the readings with the latter making it possible to determine the actual number of particles possessing a given speed and charge-mass ratio. The principal impression derived from the paper is that the phenomena are excessively complicated and depend on a multitude of almost or quite inseparable variables. Even the shape of the discharge-tube affects the relative commonness of the different kinds of charged particles (atoms and molecules, negatively and positively charged) in a drastic manner. The most striking rule emerging from the data is, that the most probable energy of the ionized hydrogen atoms stands to the total energy which would be acquired from the entire anode-cathode P.D. in a ratio (about 0.8) which is nearly independent of the variables controlling the phenomena. Comparison of the results with inferences from Doppler effect shows hardly any degree of agreement.—[R. Döpel, Munich; Ann. d. Phys. *76*, pp. 1-28, 1925.]

KARL K. DARROW

Measurements of extremely low gamma-ray wave lengths by diffraction.—Employing Bragg's method of the rotating single crystal (of rocksalt, rotated at a rate of less than 1° in 24 hours) and a photographic film to receive the diffracted rays, Thibaud succeeded in locating and measuring a line corresponding to a wave length 0.052A (236000 volts). This is attributed to thorium B; another line of the wave length 0.062 and another at 0.145 were obtained with radiothorium. Independent determinations, by measuring the speeds of electrons driven out of the *K*- level of lead atoms by the same radiation, confirmed the shortest of these wave lengths and indicated two even harder rays, as yet not detected by the crystal-diffraction method but perhaps not inaccessible.—[J. Thibaud; C. R. *180*, pp. 138-140, 1925.]

KARL K. DARROW

PHOTOGRAPHIC SENSITOMETRY WITH FLUORESCENT OILS

By George R. Harrison[1]

ABSTRACT

The changes in sensitivity and contrast obtained when Cramer Contrast Process, Seed 23, and Wellington Super-Xtreme plates are coated with any one of five different fluorescent oils were studied in the region 2200-3800 A, for the purpose of improving the precision of photographic photometry in the region 2000-2400 A, and to investigate the feasibility of extending this method into the extreme ultraviolet. Characteristic curves are given for various plate-oil combinations, and some of the changes in sensitivity and contrast found are tabulated. The effect of thickness of coating was found to be great with most oils; certain oils gave the greatest improvement in sensitivity with thick coatings, and others with thin ones. The greatest sensitivity increase was found with a clear paraffin oil and contrast plates; the greatest contrast was found with a light lubricating oil and the same plates. A 400-fold gain in exposure time by oiling was found in certain cases. Some oils were found to increase the density produced by a certain intensity if the intensity exceeded a certain value, and to decrease it otherwise. This effect varies with wave length, so that it was found possible to decrease the sensitivity at longer wave lengths and increase it at shorter wave lengths, thus making the sensitivity curve of the plate more uniform throughout the spectrum. Although the chief effect with most oils is below 2400 A, certain varieties increase the contrast even up to 3800 A, although the sensitivity was generally decreased above 2500A. Oiled plates, when the oil coating is uniform, were found to be perfectly applicable to photographic photometry, accuracy to one per cent being readily attainable even at the shortest wave lengths studied. Sections of the "characteristic surface" in which the reciprocity law holds to within one per cent were found for several wave lengths, including 2200A.

With the increasing importance of intensity measurements, especially in the ultraviolet region, new demands of accuracy and convenience have been made on photographic photometry. Fairly precise methods are now available down to wave lengths in the neighborhood of 2400A, but below this the accuracy falls off rapidly on account of the increasing absorption of the gelatine in the photographic emulsion, and the consequent decrease in sensitivity and contrast.

The writer[2] has made use of a method of photographic photometry which gives very good precision in absorption measurements, where all density comparisons are made on the same plate, for the same time of exposure, and at the same wave length. Desiring to extend this method to the extreme ultraviolet in order to attack certain intensity problems in the spectrum between 500 and 2400A, at the suggestion of Professor Theodore Lyman, he has begun an investigation of photographic characteristic curves and the precision which can be expected

[1] National Research Fellow.

[2] Harrison, Phys. Rev., *24*, p. 466; 1924.

in this region. The present paper deals primarily with the section 2400-2200A, which is a region of transition.

The average photographic plate is generally considered to have variations in the sensitivity of the emulsion of from five to eight per cent over its surface.[3] Where the gelatine content of the emulsion is reduced, as in the Schumann plate, this variation seems to be still greater, and further to be strongly influenced by humidity, seasoning, and so on. Commercial plates for the extreme ultraviolet can now be obtained, but reports as to their uniformity vary greatly. The most striking characteristic of the laboratory-made Schumann plate, at least, seems to be its tendency to "explode" in certain infected spots into high sensitivity, which renders its value doubtful in photographic photometry.

Two other means of photographing light in the region below 2000A have been described by Duclaux and Jeantet;[4] the first consists in treating an ordinary plate with sulphuric acid, which unfortunately results in an emulsion with many of the photometric difficulties of the Schumann plate. The second method makes use of fluorescent materials in which the plate is bathed, the wave length of the secondary light being such that it can penetrate a thin gelatine film. Duclaux and Jeantet used such plates down to about 1800 A, but Lyman[5] found that they were practically as sensitive as Schumann plates at least as far as 584A. Henri,[6] Hesthal and the writer,[7] and Helmick[8] have all given short accounts of the use of different fluorescent oils in regions between 1800 and 2400A. The concensus of opinion seems to be that while certain oils are more satisfactory than others, those which give the best results for one observer are frequently found to fail by another, leading to the suspicion that the principal effects are due to impurities which vary with the sample of oil chosen. That this is not really the case is indicated by the work reported on in this paper.

Since the number of possible variables is so great, exact numerical values have little meaning in a report such as the present, but an outline of the more general conclusions reached in a survey of the characteristics of several types of plate when coated with different oils will be given. Particular attention has been paid to the availability of the different

[3] Mees, Opt. Soc. Amer. Meeting, March 1925.
[4] Duclaux and Jeantet, Jour. de Phys., *2*, p. 156; 1921.
[5] Lyman, Nature, *112*, p. 202; 1923.
[6] Henri, Jour. de Phys., *3*, p. 181; 1922.
[7] Harrison and Hesthal, this journal, *8*, p. 471; 1924.
[8] Helmick, this journal, *9*, p. 521; 1924.

combinations for photometry, especially as regards uniformity and reproducibility. A new use for oils in the region 2400-3800A will be described.

APPARATUS

All characteristic-curve spectra were taken on a combination ultraviolet sensitometer and spectroradiometer with a large quartz prism, built for studying the sensitivity curves of photographic plates and for use in photochemical work. This, in the spectroradiometer form, has been described in a recent article on the mercury arc.[9] For photographic work a large plate-holder could be substituted in the place of the thermopile. This was arranged to take 10×25 cm plates, so that sixty photographs of spectra 9 cm long covering the region 2100-3800A could be taken on one plate. This gives the advantage of enabling one to compare a large number of densities which have been developed and fixed together. The vertical plate-holder was provided with a set of pins and a releasing arm at the back, so that pressing a key was sufficient to set the plate for the next exposure. Exposures were timed by an electromagnetic shutter operated by a standard clock. As a source of light a steady condensed spark between rotating cadmium disks was used, since the quartz mercury arc does not furnish satisfactory lines below 2400A.

Intensities were reduced by known amounts with the set of standard screens previously described,[2,7] calibrated on a thermoelectric densitometer. These screens are apparently truly neutral, and when kept in irregular motion constitute an accurate means of reducing intensities by known amounts in cases where the inverse square law cannot be used. The screens were hung as short pendulums, differently weighted, from a shaft bent in several places along its length, so that each screen not only swung from side to side irregularly, rebounding from rubber buffers to limit its vibration, but was also moved slightly up and down. The shaft was turned at about four revolutions per second by a small motor.

After exposure and before development the oil must be removed from the plates; various investigators have used ether and alcohol, acetone, and xylene for this purpose. Helmick[8] coated plates with anthracene dissolved in xylene, which need not be removed. While the writer got a marked increase in sensitivity with such plates, the method was not satisfactory from a photometric standpoint. Acetone

[9] Harrison and Forbes, this journal, *10*, p. 1; 1925.

seems to be the most convenient solvent when oils are used; xylene is not so easily rinsed off with water before development, and is apt to leave streaks if at all greasy, while the acetone bath can be used for some time without renewal.

All plates were developed for one minute in a standard elon-hydrochinon solution of good keeping quality. For certain purposes longer development in a weaker solution would be preferable, but this was unnecessary for photometry in this region. After being washed and dried the plates were cut into two or more pieces and measured on the thermoelectric densitometer.[10]

CHARACTERISTICS OF OILED EMULSIONS

In general, it may be said that the effect of a given variety of oil seems to be quite constant, if always used under the same conditions. The density produced by a given exposure may be expected to be influenced by the characteristic curve of the emulsion to the wave length of the incident light, due to partial penetration of the oil film, by its characteristic curve to the effective wave length of the secondary (fluorescent) light, and by the "fluorescence characteristic curve" of the oil. The last is a convenient expression for the relation connecting the intensities of incident and fluorescent light, and is affected by a number of factors, including the absorption of the oil for all wave lengths concerned, the thickness of the oil layer, and the mechanics of fluorescence in the oil molecule. It is apparently not a linear function, and varies with the variety of oil used.

A number of different oils were tested roughly, and finally the five found most useful were carefully studied on three types of plate: Cramer Contrast Process (transparency), as a typical slow plate; Seed 23, as medium; and Wellington Super-Xtreme as fast. It has long been suspected that speed ratings of plates, made with white light, have little meaning in the ultraviolet region; in fact, in many cases the "slower" plates seem to have greater speed. One would expect, in any case, that the best combination would be obtained by using an oil with strong absorption for all the important incident wave lengths, transforming them efficiently into light of the proper wave lengths to be near the maximum sensitivity region of the plate, with a plate having the desired characteristics in its region of maximum speed.

In determining characteristic curves, the lower half of the plate was coated with the oil under investigation, and the upper half left bare.

[10] Harrison, this journal, *10*, p. 157; 1925.

Inasmuch as the thickness of oil layer was found to have a decided effect on sensitivity, two sets of spectra for characteristic curves were always taken on each oiled portion, one with the highest intensities at the bottom, and the other with them at the top. These could be averaged, so as to offset spurious contrast effects brought in by variable thickness. Then the desired spectra were taken on the uncoated portion of the plate, and it was washed in acetone, rinsed, developed, fixed, and dried. A rather wide slit was used in the sensitometer so that each line was represented by a rectangle 1×3 mm in size.

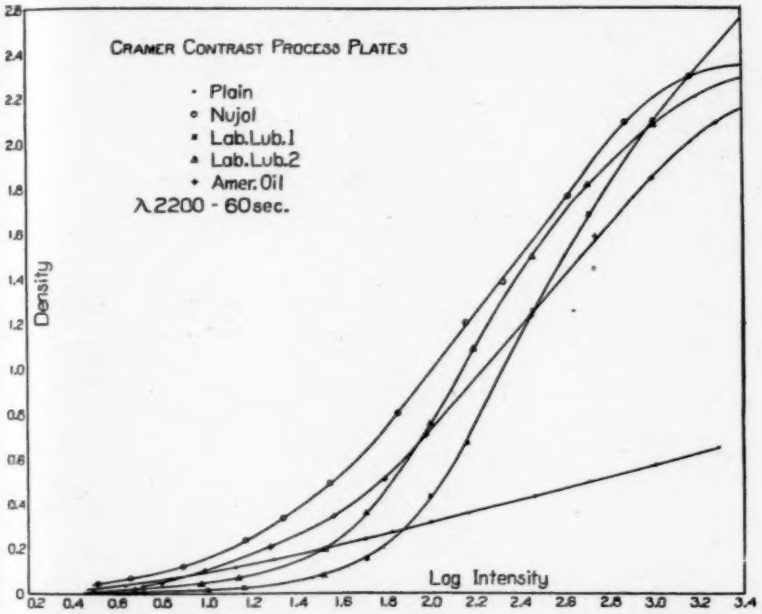

FIG. 1. *Characteristic curves for Cramer Contrast Process (transparency) plates when dry, and when coated with four different fluorescent oils, exposed to light of wave length 2200A for 60 seconds.*

In Fig. 1 are given characteristic curves for Cramer Contrast Process plates unoiled, and when coated with four different oils: Nujol, a commercial paraffin oil, clear white, with no visible fluorescence and an absorption band beginning at 2400A and extending downwards, with no appreciable absorption at longer wave lengths; American oil, a commercial paraffin oil similar to Nujol but slightly heavier, having a similar but less marked absorption band; Lab. Lub. 1, a light machine

oil of a brownish color with a green tinge in sunlight, which showed a brilliant blue fluorescence under ultraviolet light, and completely absorbed the whole ultraviolet region; and Lab. Lub. 2, a very light, clear machine oil, with a slight bluish tinge, and a faint green fluorescence in sunlight, absorbing everything below 3000A in the region investigated. Russian oil, a third brand of paraffin oil, was also studied, but the results lay between those for American oil and Nujol, and will not be discussed.

All data for Fig. 1 were taken with an exposure time of 60 seconds, which was chosen as the value most used by the writer in photometric work. Although differences in behavior would appear if greatly differing times of exposure were studied, the approximate validity of the reciprocity law in this case, noted below, makes it apparent that these would be small in the range of time and intensity commonly used. It is evident from the curves that where high sensitivity is desired, if a Cramer Contrast Process plate is used, Nujol is the best oil at all intensities except the very highest, where Lab. Lub. 1 produces the greatest density. Lab. Lub. 1 also gives the greatest contrast. It will be noted that below density .29 a plate coated with Lab. Lub. 1 is less sensitive than an uncoated plate. This may explain the failure of certain investigators to get good results with oiled plates, especially since these curves are for only one time of exposure, and the intersection of the two curves varies somewhat with time, and greatly with wave length and emulsion used. This absorptive action on light of low intensity suggests an important use of oiled plates in increasing contrast. It is possible to take a spectrum photograph on a plate which normally shows strong lines blending diffusely at the edges into a fairly strong continuous background, and with a plate oiled with Lab. Lub. 1 to practically wipe out the background and make the lines still stronger.

In Fig. 2 are shown characteristic curves of Lab. Lub. 1-oiled Cramer Contrast Process plates at four selected wave lengths, the solid curves being for the dry plates, and the dotted curves for the oiled plates. On each pair of curves the point of intersection has been marked with a square; below this point a given light intensity will produce a greater density on the dry plate, and above it on the oiled plate. The relative positions of the different *pairs* of curves on the log I axis have no meaning. It is evident that the greater the wave length, the higher the density above which oiling increases density, so that at 3350A and above it probably never does so. Hence, it is possible by using oil to so weaken the lines in the longer wave lengths, and to so strengthen

those in the shorter wave lengths, that a fairly uniform spectral density distribution is obtained, even with a source more intense at longer wave lengths, and plates normally more sensitive there. Nujol, on the other hand, has practically no effect on wave lengths greater than 2400A, and greatly strengthens those below, so that it is conceivable that by the proper mixing of certain oils many results useful in photographic photometry could be obtained. In particular, it is desirable that where absorption measurements are being made throughout the spectrum, approximately similar densities be obtainable at all important wave lengths for the same time of exposure.

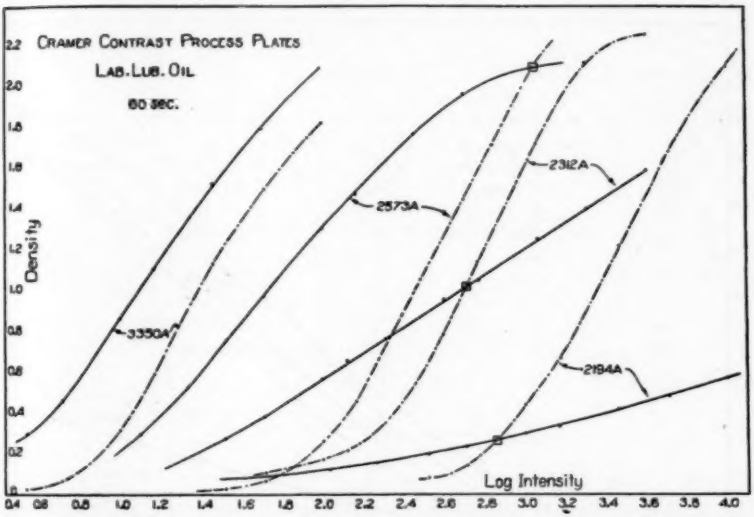

FIG 2. *Characteristic curves for Cramer Contrast Process (transparency) plates when dry and when coated with Lab. Lub. 1 oil, at four different wave lengths. The curves for the different spectral regions have been displaced along the log I axis relative to one another.*

In Fig. 3 are shown characteristic curves similar to those in Fig. 1, for the same oils used with Seed 23 plates at 2200A. This wave length is of interest because the sensitizing action of the oils is most extreme there, of all wave lengths studied. Indications point to the probability that the effect will be still greater at shorter wave lengths, for some distance, at any rate. It is evident that the contrast obtained with the Process plates is not to be expected with Seed 23 plates when coated with oil. The extra fast plates gave very poor results when oiled; in fact, Nujol actually reduced the sensitivity.

In Table 1 are listed the various plate-oil combinations found most interesting for investigation; the third column gives the contrast at the straight portion of the curve for one minute exposure to λ2200A; the fourth gives the intercept of the straight portion of the curve on the log I axis, which is a measure of the inertia, and hence inversely of the speed of the combination; the fifth column gives a figure of merit which is inversely proportional to the relative intensity of light which would have to be used to produce unit density under the conditions stated. This is also a measure of the advantage of the oil in ordinary spectrum photography where reducing the time of exposure is the principal aim,

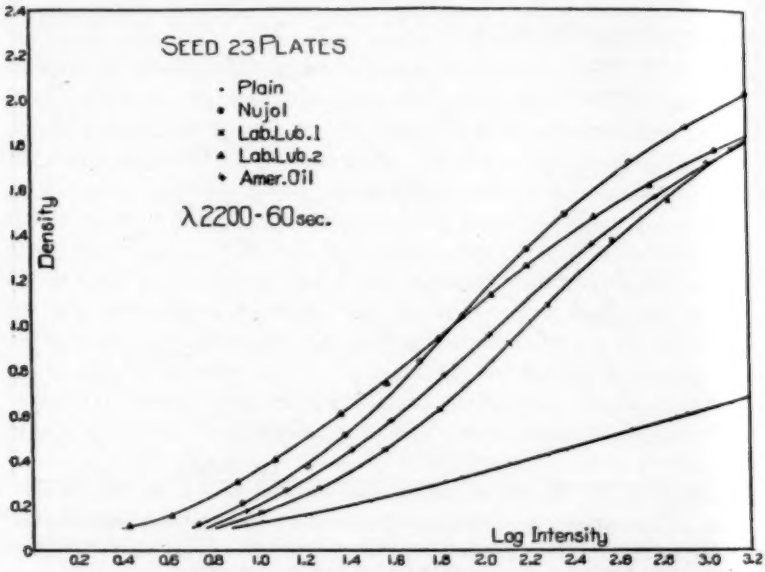

FIG. 3. *Characteristic curves for Seed 23 plates when dry, and when coated with the same oils used in Fig. 1.*

since the reciprocity law may be assumed in this case for all practical purposes. The figure of merit of each combination will of course vary greatly with the density to be produced, the wave length, and so on, and is referred to unity for each variety of dry plate. When more data is available on the absorption and fluorescence curves of different oils, and on the sensitivity curves of photographic plates, better plate-oil combinations will doubtless be found. For the present the use of either

Nujol or Lab. Lub. 1 with a very contrasty plate is found quite satisfactory, the choice of oil depending on the result desired.

UNIFORMITY TESTS

In order to measure the amount of error to be expected, a plate was dipped in Nujol over half its length, and allowed to drain for five minutes. The surplus oil was then wiped off the bottom, and the plate allowed to drain for thirty minutes, when ten exposures of 20 seconds each were taken with the spectrum of the cadmium spark on the oiled portion, and ten on the dry portion. When this plate was measured it was found that about four times the opacity was obtained where the

TABLE 1. $\lambda = 2200A$ *Time = 60 seconds*

Plate	Oil	Contrast	Intercept	Relative merit for d=1
Cramer Contrast Process	Lab. Lub. 1	1.83	2.01	191
—	Lab. Lub. 2	1.62	1.74	290
—	Nujol	1.30	1.45	400
—	American Oil	1.14	1.58	230
—	None	.26	1.00	1
Seed 23	Lab. Lub. 1	.97	1.22	138
—	Lab. Lub. 2	.80	.64	290
—	Nujol	1.11	.92	290
—	American Oil	.90	.98	182
—	None	.29	.74	1
Wellington Super-Xtreme	Lab Lub. 1	.39	1.22	3.8
—	Nujol	.35	1.20	2.0
—	None	.38	1.72	1

oil was thick as where it was thin, so that, at least up to one-fourth millimeter, perhaps, the thicker the coating of paraffin oil the greater the sensitizing action. This is because the region 2200-2400A is only on the edge of an absorption band, and absorption is not complete in a thin layer. When similar experiments were tried with the other two paraffin oils, analogous results were obtained. With Lab. Lub. 1, however, the effect was reversed; this had such a very strong absorption coefficient for both $\lambda 2200$ and the fluorescent light that the thinner the coat the better. These results emphasize the point that if accurate photographic photometry is to be possible, the oil thickness must be uniform. In further uniformity tests the plates were either dipped and drained for a long time, so that a slowly and *uniformly* varying layer resulted, or a very thin coating was applied with a bit of chamois. A

plate prepared in this way was tested for uniformity, both of the dry emulsion and of the oiled surface. To measure the accuracy of the densitometer, one fairly dense rectangle was left over the slit and measured ten times without removal. This rectangle was then measured ten times, being removed and replaced between each measurement. Next each one of the ten λ2200 rectangles on the dry portion of the plate was measured ten times, and finally those on the oiled portion were similarly treated. The following results were obtained for the dry plate:

Error from	Average deviation from mean of ten determinations. (Per cent)	Maximum deviation from mean of ten determinations (Per cent)
Densitometer	.11	.16
Densitometer plus zero drift (due to neighboring spark and electric fan)	.20	.47
Resetting same rectangle (due to unevenness)	.56	1.01
Total error (due to spark, emulsion variation, timing, and all above sources)	1.77	2.50

The surprisingly small total error for the dry plate, in which emulsion sensitivity variations can probably be blamed for only about two per cent error, is doubtless due to the fact that in this method of photographic photometry, all densities compared are in the same vertical line, and are thus the same distance from the edge of the plate. This, next to the advantage of having all wave-lengths on the same plate, and all densities developed together, is probably its greatest asset.

When the oil-coated surface was similar tested, the errors were found to be no greater. Where care in coating was used, the maximum deviation was 2.4 per cent, including all errors listed above, and a great gain in sensitivity was obtained.

RECIPROCITY LAW TEST

In working with wave lengths much shorter than 2000A, where a vacuum spectrograph is necessary, it is difficult to vary the light intensity by known amounts at all wave lengths. Accordingly we must have recourse to the reciprocity law, or Schwarzschild's law, or any other such definite relation that can be expected to hold more or less accurately in that region. For this reason a preliminary test was made

of the reciprocity law at λ2200, and the results given in Table 2 were obtained. In this case, the reciprocity law was found to hold with an average deviation from the mean of 1.14 per cent, and a maximum deviation of 2.4 per cent, of which two-thirds at least is due to emulsion, coating, and setting errors, the systematic deviation being very small. At wave length 2573 on the same plate the reciprocity law held with an average deviation of one per cent and a maximum of 1.8 per cent, while there was no regular deviation. The dry plate, on the other hand, showed a small deviation from the reciprocity law under the same conditions.

TABLE 2. *Test of Reciprocity Law*
Cramer Contrast Process Plate—Lab. Lub. 1 Oil—λ2200A

Intensity	Time	Product $I \times t$	Densitometer Reading	Corrected Reading	Deviation from Mean
100	5 sec.	500	33.6	33.6	+.39
51	10	510	33.9	33.2	−.01
29	17	493	33.2	33.7	+.49
15	33	495	33.6	34.0	+.79
10	50	500	33.4	33.4	+.19
5.1	98	500	32.8	32.8	−.41
3.3	150	495	32.5	32.8	−.41
			Mean	33.21	.38

Ave. per cent deviation from Mean 1.14
Max. per cent deviation from Mean 2.38

It may be concluded, then, that oiled plates are entirely satisfactory for photographic photometry at short wave lengths, and that by using them results to within one per cent can be readily obtained for measurements of the kind ordinarily carried out by photographic methods, such as the determination of absorption coefficients.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY,
MARCH 23, 1925.

Observations on Electrons recoiling from Impacts of Quanta.—Using the C. T. R. Wilson expansion-method upon air traversed by x-rays (pulsating and not quite homogeneous, although carefully filtered) the authors observe the tracks of the released electrons, and find themselves able to discriminate between the "short tracks" of electrons recoiling from impacts of quanta after an interchange of momentum and energy, and the "long tracks" of electrons ejected from atoms owing to the total absorption of quanta in these atoms. The latter, they find, are predominant at voltages of the order of 20 kv;

the former are almost the only ones present at voltages of the order of 100 kv. Counting all the tracks produced in individual cases, identifying those of the two kinds (evidently they are rarely in doubt) and computing the ratio of the number of recoiling electrons to the number of ejected or "photo" electrons, they find that this ratio agrees fairly well with the ratio of total scattered intensity to total absorbed intensity, as determined by independent prior experiments on the loss of energy by x-rays traveling through air. Determining the lengths of the longest tracks of recoiling electrons, at four voltages between 50 and 111 kv, they find a very good agreement with the ranges to be expected for electrons which have been knocked directly forward by a central impact of the impinging quantum. Even the distribution-in-angle of the initial directions of motion of the recoiling electrons agrees, so far as it is possible to test it, with the special theory devised by Compton and Hubbard to cover this quantity.—[A. H. Compton and A. W. Simon, Chicago; Phys. Rev. *25*, pp. 306-313; 1925.]

KARL K. DARROW

Visualization of the Disintegration of Nitrogen Nuclei by Impacts of alpha-particles.—Out of 270,000 alpha-particle tracks photographed by the C. T. R. Wilson expansion-method, eight ended in forks of which one branch is short and stubby in appearance, while the other is a long fine straight line. The length and thinness of the latter are such as would be expected for the track of a proton, a particle of unit charge and great speed; the appearance of the other suggests a particle as massive as the nitrogen nucleus, or more so. As there is no third track, one must assume that the alpha-particle and the residuum of the nitrogen nucleus have adhered together in a new system. Assuming this, and assuming that momentum is conserved in the impact while kinetic energy is not, Blackett calculates values for the speed of the proton which are conciliable with the known ranges of protons produced in the same fashion and observed by the scintillation-method; and values for the speed of the newly synthesized particle which, when compared with the length of its visible path, indicate that its mass is indeed a little greater than that of a nitrogen nucleus. The kinetic energies of the two particles add up to a sum which in five cases out of six is less than the original kinetic energy of the alpha-particle, (the values however are very scattering). The newly synthetized particle therefore would be an isotope of oxygen, of mass 17 and greater internal energy than the nitrogen nucleus.—The number of these forks observed among the 270,000 trails, while small, is still several times greater than would be expected if the inverse square law continued to be valid down to indefinitely close approaches of the alpha-particle to the nitrogen nucleus. The photographs are magnificent.—[P. M. S. Blackett (Cavendish); Proc. Roy. Soc. *A107*, pp. 349-360, 1925.]

KARL K. DARROW

JOINT MEETING OF THE OPTICAL SOCIETY OF AMERICA WITH THE AMERICAN PHYSICAL SOCIETY

MINUTES

A joint meeting of the Optical Society of America and the American Physical Society was held at Columbia University in New York City on Friday and Saturday, February 27 and 28, 1925. The session on Friday morning was conducted by the Optical Society, and the session on Saturday afternoon by the Physical Society; the sessions on Friday afternoon and Saturday morning were joint meetings of the two societies. At the joint meeting on Saturday Dr. W. T. Bovie of Harvard University gave a most interesting lecture on "The Eye as a Light-Sensitive Instrument." Dr. C. E. K. Mees was to have delivered a lecture at this same session on "The Properties of Photographic Materials in their Relation to Uses in Physical Measurements;" but, as he was not able to be present in person the lecture was read for him by Mr. L. A. Jones. Professor M. Siegbahn was to have delivered a lecture on Friday afternoon on "X-Ray Spectra and Optical Spectra," but was prevented by illness from attending the meetings.

The following are titles and abstracts of papers and reports presented by the Optical Society. Papers marked with an asterisk (*) in the session on Friday afternoon were contributed by the American Physical Society.

J. P. C. SOUTHALL, *Secretary pro tem*

FRIDAY MORNING

Session of The Optical Society of America

1. **Charles Bittinger,** *Chairman* — **Brookline, Mass.**

REPORT OF COMMITTEE ON COLOR TERMINOLOGY QUESTIONNAIRE

2. **L. A. Jones,** *Chairman* — **Eastman Kodak Company**

REPORT OF COMMITTEE ON PHOTOGRAPHIC STANDARD OF INTENSITY

3. **H. E. Merwin and George W. Morey** — **Geophysical Laboratory**

THE OPTICAL PROPERTIES OF THE GLASSES OF THE TERNARY SYSTEMS Na_2SiO_3-$CaSiO_3$, AND K_2SiO_3-$CaSiO_3$-SiO_2, AND THE INFLUENCE OF SMALL AMOUNTS OF IRON

A compilation of the available data on the properties of the alkali-lime glasses showed an unfortunate lack of concordance, probably as a result of making highly accurate measurements of physical properties and guessing at the chemical composition. The above two systems were studied fairly completely, using glasses made by melting the pure components in platinum, making sure of homogeneity by pulverizing the glasses and determining the refractive index under the microscope by the immersion method. The homogeneous glasses were analyzed and their dispersion determined by the prism method. The results will be presented in summary.

(The present paper will appear in full in the *Journal of the Optical Society.*)

4. **K. S. Weaver** — **Westinghouse Lamp Company**

A DETERMINATION OF REFLECTION FACTORS OF PRINTED COLORS

By using a spectrophotometer in connection with a Taylor Reflectometer a considerable reduction in labor was obtained in the determination of the reflection factors of a series of about eleven colors printed on white paper in twelve different intensities.

The spectrophotometer is used on one sample of each color group and the data so obtained are used to correct the Taylor Reflectometer readings which are taken on each sample.

The method gives results corresponding to those obtained on the spectrophotometer with time expenditure corresponding more nearly to that required by the use of the Taylor Reflectometer.

One set of data obtained in the application of this method may be used to obtain reflection factors for incident light of any color.

Bibliography:
A Simple Portable Reflectometer of the Absolute Type, A. H. Taylor, Journal Optical Society, Jan. 1920.
The Visibility of Radiant Energy, K. S. Gibson & E. P. Tyndall, Trans. I.E.S., Feb. 1924.

5. G. W. Moffitt and O. K. Kaspereit — **Frankford Arsenal**

THE RESIDUAL ABERRATIONS IN COMA-FREE CEMENTED OBJECTIVES OF THE CROWN-LEADING TYPE

A comprehensive survey of the properties of telescope objectives of this type has been made, and systematic data on the variations of the residual aberrations caused by the use of different glasses have been obtained. Not only are the results of value to the designer, but they also indicate the directions in which it would be advantageous to go, if possible, in the production of new glasses.

(Received January 21, 1925)

6. I. C. Gardner — **Bureau of Standards**

A MAGNIFYING STEREOSCOPE AND CAMERA LUCIDA: TWO INSTRUMENTS FOR AIRPLANE MAPPING

A description of two instruments designed and built at the Bureau of Standards for the use of the Coast and Geodetic Survey. The stereoscope is used to locate on airplane photographs the landmarks used in the reconnaissance survey which are often inconspicuous when viewed from above. The camera lucida enables one to rectify an airplane photograph, i.e., eliminate the distortion arising from the pitch and roll of the plane, and reduce to a standard scale without making a photographic copy of the photograph.

7. I. C. Gardner and A. H. Bennett — **Bureau of Standards**

INTERFERENCE EFFECTS AND THE HARTMANN TEST

A modification of the Hartmann test is described which permits the spherical aberration to be measured as the difference of phase, expressed in wave lengths, of the different zones of the wave-front on arrival at the focus. No apparatus is required other than that ordinarily available for the Hartmann test.

8. William G. Exton, M.D. — **Laboratory of Prudential Life Insurance Company**

A NEW AND DIRECT METHOD OF MEASURING THE CLOUDINESS OF LIQUIDS (SCOPOMETRY)

In protein studies made in the Prudential Laboratory it was found necessary to overcome difficulties and errors involved in handling two suspensions for each determination such as nephelometry demands. Experiments led to the development of an instrument called the Scopometer which gives direct readings instead of comparisons.

The design of the Scopometer utilizes a new principle. Light is admitted through a narrow slit or target placed below the bottom of a cup holding the cloudy liquid. This incident light serves as an image which becomes extinguished by varying the light scattered or reflected from the particles in suspension and also by other effects such as massing the particles. As designed, the scopometer is an optical train consisting from above downward of an eyepiece; an optical plunger on rack and pinion; a cylindrical cup with plano-parallel bottom holding the liquid to be examined and target below which is a mirror which reflects light through the target (of platinized glass having a clear slit of predetermined width) Standard illumination with controls and a condensing system giving parallel beams is admitted through a shutter

on the front of the instrument which works on a rack and pinion so as to mark the height of the beam of light which falls on the long axis of the cylindrical cup holding the cloudy liquid.

Operation of the instrument is exceedingly simple and rapid and is carried out in several different ways, i.e., by raising and lowering the plunger with open or closed shutter; or by fixing the plunger and raising or lowering the shutter. All these arrangements permit readings so fast that errors of settling are eliminated and also have the advantage of affording a distinct endpoint instead of matching brightness as in turbidimetry and nephelometry. With the optical train as a constant, differences in plunger readings and turbidities remain the only variables, and the height of the plunger (column of liquid) which extinguishes the target bears a definite and simple relation to the turbidity. Work done in the Prudential Laboratory shows that plunger readings on a vernier scale correspond regularly with protein concentrations expressed in milligrams per hundred c.c., and regular curves for different set-ups have been obtained which give a total range extending from optical clarity to opacity. Thus by calibration for a given determination Scopometry permits the translation of subsequent results directly into terms of concentration with freedom from errors hitherto inherent in the preparation, manipulation and matching of suspensions. Other uses of the Scopometer include water and dust analyses and counting objects like bacteria, blood cells, etc.

The present paper will appear in full in Journal not yet decided upon—Probably Journal Industrial & Engineering Chemistry or Journal of Am. Chemical Society).

(Received January 21, 1925)

9. Wm. Mayo Venable **Blaw Knox Company, Pittsburgh**

THE QUANTUM THEORY AND THE STIMULI FOR THE VISUAL SENSATIONS

The sensations which we call hues of the spectrum are composites of brightness sensations white, yellow and green, with darkness sensations black, blue and red or purple, as shown in a former communication to this society. The wave lengths at which distinctive colors may be seen in the spectrum at least intensity are those at which these elementary sensations are severally most effectively excited. These are: white 507, yellow 565, green 535, red 632, orange 611, blue 496, violet 443 and indigo 462 millimicrons. If the molecule of Visual Purple be constructed upon the general plan of the Bohr atom, absorption of energy at frequencies corresponding to these wave lengths will take place most readily, and will take place in quanta. It is found that the requirements of the quantum theory are met with great precision if there are electrons expelled from molecules of visual purple, from a normal position in the 12th shell, to certain outer orbits. Assuming that those electrons that arrive in only certain outer orbits defined by the wave lengths above specified are able to bring about a reaction resulting in visual sensation, and that there is a reaction between molecules of visual purple with electrons in certain orbits only and the nerve cells in the retina (a reversible chemical reaction), it is shown that the requirements of the quantum theory are met, so far as energy transitions are concerned, if there is a single quantal reaction between the purple and the nerve. Designating this quantum of energy by x the stimulus for the sensation white is 6x, for yellow 3x for green 2x, for red or purple $-2x$, for blue $-3x$, for black $-6x$. These reactions are precise. No other size of quantum associated with nerve reaction is necessary to explain stimuli for all varieties of color sensations.

The reactions, as given in the tables, are of the same order of accuracy as the series relationships of spectral lines.

The present paper will appear in full in *American Journal of Physiological Optics*.

10. Arthur Pope **Harvard University**

THE DISTINCTION BETWEEN THE ATTRIBUTES OF CHROMA AND OF SATURATION AND BETWEEN THOSE OF BRILLIANCE (OR VALUE) AND OF BRIGHTNESS (OR INTENSITY OF VIBRATION).

1. In books like *Color and its Applications* by Luckiesh and *Colour* by Martin and Gamble, the term *saturation* is used in two different senses, sometimes as the equivalent of *chroma*, as

defined by Munsell, and sometimes for a distinct attribute of tone or color. The existence of *saturation* as a distinct attribute has not been recognized by scientists with the result that there has been much unnecessary confusion.

2. There is similar confusion in use of term *brightness* sometimes as equivalent of *value* or *brilliance*, sometimes as referring to another attribute to be distinguished from *value* or *brilliance*.

3. The relation of *saturation* and of *brightness* to *value* and *chroma* may be expressed in the tone or color solid. In any one *hue*, lines of uniform *value* are horizontal; lines of uniform *chroma* are vertical; lines of uniform *saturation* are diagonal converging on black; lines of uniform *brightness* are also diagonal.

4. The existence of these two additional attributes of tone or color should be considered in connection with the adoption of any system of nomenclature.

(Received January 22, 1925)

Bibliography:
Munsell, A Color Notation.
Luckiesh, Color and its Applications.
Martin and Gamble, Colour.

11. Christine Ladd-Franklin — **Columbia University**

PUTTING PHYSIOLOGY, PHYSICS AND PSYCHOLOGY TOGETHER

The reason that *comprehension* of the vast mass of facts already well known regarding the chromatic sensations has made practically no progress since the days of Helmholtz in that these *acts* are somewhat equally distributed among three different sciences, and that consequently there is no one who is able (or at least willing) to hold them all in hand at once. (1) Important work by Hecht has been appearing in the last six volumes of the Journal of General Physiology which makes it quite certain that the initial retinal process is a photo-chemical one and that the light-sensitive substance concerned in the primitive white of the rods is the same substance (as shown by its subjective-intensity-distribution) as that in the cones. (But Hecht here overlooks the fact that the circumstance that the *sensation* is the same, that of whiteness,—the basis of my color-theory—is already an irrefragable proof of this identity.) (2) Most important of all of these contributions is the fact that König determined (1892) the actual constituents of the tri-receptor retinal photo-chemical processes by means of establishing (after proper change of the independent variables) perfect coincidence between normal tetrachromatic vision and that of both of the two types of atavistic dichromatic vision. This is a fact, however, that, by a marvellous chain of accidents, has been overlooked by the physicists themselves. I have endeavoured to restore it to its proper significance in the Appendix which the Optical Society has permitted me to write to the English translation of Helmholtz. (3) The contribution of Psychology to this subject is (a) the laying down (1888) by G. E. Müller of the fundamental principles that must govern the formation of any theory concerning the neuro-psychic correlations, and (b) the work done by Hering during his fifty years of magnificent activity (aided by a large body of students both at Prag and at Berlin) in showing successfully that the so-called theory of Helmholtz is no theory at all. His own theory, however, exists only by denying the great Helmholtz *fact* that the initial photo-chemical process is a tri-receptor process (I am much pleased to see that Howell, in the 9th edition of his Physiology, just out, has adopted all my terminology). In neglecting Hering's *theory*, however, the physicist has unfortunately overlooked the *fact* which made it a necessity —the fact that both *yellow* and *white* are simple unitary *sensations*, no matter what their *physical* basis. In conclusion I permit myself to quote from a letter just received from M. Piéron of the Collège de France: "Il est certain que votre théorie concilie le mieux les faits si complexes relatifs au fonctionnement lumineux et chromatique."

(Read by title)

(Received January 21, 1925)

12. Richard Hamer — **University of Pittsburgh**

PHOTOELECTRIC THRESHOLDS OF THE SECOND GROUP AND THE QUANTUM THEORY

A comparison of the photoelectric thresholds, V_0, of the elements of the second group and the spectroscopic ionizing energies for electrons in the 2P orbits, I_{2p}, show a remarkable and consistent agreement in the cases of *Mg, Ca, Zn,* and *Cd*. *Mg.* V_0 3·23, I_{2p} 3·285; *Ca* V_0 3·08 ± .16, I_{2p} 3·167; *Zn* V_0 3·60 ± .08 I_{2p} 3.582; *Cd* V_0 3-43 I_{2p} 3·560 Volts. Assuming exact agreement the threshold values for Be, Sr, Ba, Hg, and Ra can be predicted. A linear relationship seems to hold for the left sub-group. Agreements are presented in favor of the view that the electrons occupy an electronic lattice interpenetrating the ionic lattice and one in which the threshold energy $h\nu_0$ equals the limiting minimum potential energy for electron so that prior absorption is not necessary. It is not necessary to assume a supernatant atmosphere of free atoms. The irregularities of the surface layer present portions of the boundary of the electronic lattice where the potential energies of the bound electrons vary more or less continuously from the minimum approaching the spectroscopic I_{2p} value to that for an electron completely surrounded by an infinitely extending lattice. This would account for the continuous rise in the photoelectric current for radiation of greater energy than that at the threshold.

Bibliography:
A. L. Hughes, Bulletin on Photoelectricity and Ionization Potentials.
Foote and Mohler, The Origin of Spectra.
Landolt-Bornstein's Tables 1923.
K. T. Compton and Mohler, Critical Potentials Bulletin No. 48, N. R. C.
R. Hamer, "Photoelectric Thresholds etc." Journal Opt. Soc. Vol. 9, No. 3, Sept. 1924.

13. Richard Hamer — **University of Pittsburgh**

THE PHOTOELECTRIC THRESHOLD OF AN ANIMAL RETINA

Two retinas from the eyes of a freshly killed calf were mounted on thin bare sections of microscopic cover glasses about 4 cms by 1 cm and dried under cover in air. The upper part of the retina was laid over a thin strip of copper which served to make electrical contact with a string electrometer. The retinas so mounted were suspended in a photoelectric cell and after being in a vacuum produced by a Gaede rotary mercury pump for several hours were tested for a photoelectric effect. No measurable current was observed with visible or ultraviolet light having a wave length greater than about 3000 a.u. The apparatus was the same as that used by the author for the determination of photo electric thresholds and described in the September issues of the *Journal of the Optical Society*, 1924.

Bibliography:
Journal Optical Society, Sept. 1924.
"Photoelectric Thresholds of Elements Under Ordinary Conditions," R. Hamer.

FRIDAY AFTERNOON

Joint Session of the Optical Society of America and The American Physical Society

1. Wm. Mayo Venable — **Blaw Knox Company**

THE STATIC ATOM

The spectrum of Hydrogen can be explained fully as due to absorption and emission of energy attending the formation of molecules of various sizes, as represented by H_1, H_2, H_3, H_4, etc., without resorting to a theory of multiple orbits for a single atom. If the atom and the molecule be regarded as capable of static stability, rather than dynamic stability, the fundamental equations of classical mechanics may be applied and the frequencies characteristic of spectral lines fully explained as due to the natural period of vibration of particles

about a mean position of stability. No discontinuity postulate is necessary. This conception of the atom and the molecule makes possible an explanation of nuclear structure which accounts for atomic weights and balances, and provides a model which may be used by the chemist as well as by the physicist. The conception is shown to be in keeping not only with our knowledge of the spectrum of Hydrogen, but also with what is known about the spectra and energy absorptions of Helium, Neon, and the seven elements of the first series of the periodic table. Diagrams showing the relative positions of the electrons and protons in the nuclei of these elements are shown.

2. Deane B. Judd — **Cornell University**

CHROMATIC VISIBILITY COEFFICIENTS BY THE METHOD OF LEAST SQUARES

Taking the three elementary color excitation curves, weighting them according to their brilliance, and summing them for each wave length, an approximation to the visibility function should result. The weights for the excitations which produce the closest approximation to the visibility function are taken as the most probable values for the true chromatic visibility coefficients.

To find these values, the condition was imposed that the squares of the errors between the actual observed visibility and the approximate visibility by summation of the weighted excitations should be a minimum when summed over the entire wave length range. The values of the weights which satisfy this condition have been calculated as follows:

Chromatic Visibility Coefficients

	I.E.S. Visibility	Gibson-Tyndal Visibility
Red Excitation	0.455 ± .019	0.438 ± .015
Green Excitation	0.538 ± .018	0.547 ± .014
Blue Excitation	0.00691 ± .0077	0.0151 ± .0060

The Gibson-Tyndall data is shown to be distinctly in better agreement with the color data (Probable error—0.77 per cent) than is the Standard I.E.S. and O.S.A.Visibility, (Probable Error—0.99 per cent).

Some of the uses of the coefficients in the transformation of color data from one system of specification to another are discussed.

Bibliography:
Troland, L. T., Report of the Committee on Colorimetry for 1920-21, J.O.S.A., 6, p. 548, August 1922.
Ives, H. E., The Transformation of Color-Mixture Equations, J. of Franklin Inst., 180, December 1915.
The present paper will appear in full in *Journal of the Optical Society of America and Review of Scientific Instruments.*

(Received January 19, 1925)

3. K. S. Gibson — **Bureau of Standards**

SOME TESTS ON THE ACCURACY OF MEASUREMENT WITH THE ROTATORY DISPERSION COLORIMETRIC PHOTOMETER

In December, 1923, a paper was published in the Journal of the Optical Society by Irwin G. Priest on "The Colorimetry and Photometry of Daylight and Incandescent Illuminants by the Method of Rotatory Dispersion" in which were described in detail the theory, design, and method of use of the rotatory dispersion colorimetric photometer now in use at the Bureau of Standards for the color grading of light sources having colors matched by a Planckian radiator, real or hypothetical, and for the measurement of the relative intensities of such sources under conditions of exact color match, thereby eliminating the usual uncertainties of heterochromatic photometry.

Since its installation, the instrument has been extensively used for the determination of color temperature, especially of artificial daylight units, but only recently has occasion arisen to make intensity measurements. Incidental to the work which has thus been done, certain checks on the accuracy of the measurements have been obtained. At the request of Mr. Priest

it is planned to publish these data which, while in no sense exhaustive, yet constitute a rather rigid test of the performance of the colorimetric photometer.

The accuracy thus attained with the instrument will be illustrated at the meeting.

Bibliography:
J.O.S.A. & R.S.I., 7, 1175; December, 1923.

The present paper will appear in full in *Journal of the Optical Society of America and Review of Scientific Instruments.*

(Received January 21, 1925)

*4. **W. H. McCurdy** — **Princeton University**

THE REFRACTIVE INDEX OF AN EXCITED GAS

5. **A. G. Worthing** — **Nela Research Laboratory**

POLARIZATION AND DEVIATION FROM LAMBERT'S COSINE LAW FOR LIGHT EMITTED BY TUNGSTEN, TANTALUM, AND MOLYBDENUM AT INCANDESCENCE

With a disappearing filament pyrometer, brightnesses at various angles of emission, ϕ, have been measured at $\lambda = 0.665\mu$ for the natural radiation and for its two polarized components B_s and B_p. As ϕ increases from 0° to 90°, B_p decreases at first slowly and then more rapidly to zero. B_s, however, increases in each case to and passes through a maximum at about 80° which is about double the normal value, and then decreases very rapidly to zero. With B_s and B_p expressed in per cent of the values for normal emission, the curves for B_s and B_p as functions of ϕ change but little if at all with temperature. The total brightness $B_s + B_p$ similarly to B_s passes through a maximum of from 115% to 120% of normal at about 75°. The average brightness for tungsten taking account of all directions is about 4.5% greater than normal brightness, for tantalum about 4.0% greater, for molybdenum about 6.0% greater.

The polarization with increasing ϕ, in each case, changes from zero at first slowly and then more rapidly toward unity at 90%, though complete polarization is not attained. For straight filaments of circular cross-section, the light emitted in a direction normal to the length of the filament is about 20% polarized.

Bibliography:
Astrophysical Journal, Vol. XXXVI, No. 5, Dec. 1921.

(Received January 21, 1925)

6. **Ernest Merritt** — **Cornell University, Dept. of Physics**

SPECTROPHOTOMETRIC MEASUREMENTS OF CERTAIN IRIDESCENT COLORS

Spectrophotometric measurements have been made of the light reflected at different angles from the neck feathers of a pigeon (brilliant blue green), a butterfly wing (blue), and different samples of blue steel. In each case the spectrum of the reflected light shows an absorption band, or rather a broad region of small intensity, which shifts toward the shorter waves as the angle of incidence increases. Qualitatively this is in agreement with the view that the color effects are due to interference in thin films. If it is assumed that the film has a thickness e and an index of refraction we should have

$$\frac{k^2}{4e^2} = n^2 - \sin^2 i$$

where λ is the wave length of maximum destructive interference, i.e., of minimum apparent reflecting power, for a given angle of incidence i, and k is an integer. If λ^2 is plotted against $\sin^2 i$ we should therefore obtain a straight line and straight lines were in fact obtained in each of the cases studied. It was clear from the general form of the spectral distribution curve

that k must be taken equal to one. The values of e and n computed from the intercepts of the line, were as follows:

Blue steel	$e = .154\mu$
	$n = 1.98$
Purple steel	$e = .136\mu$
	$n = 1.91$
Butterfly wing	$e = 0.19\mu$
	$n = 1.65$
Feather	$e = 0.22\mu$
	$n = 1.43$

The results confirm the view supported by Mason that all of these colors are due to films.

The present paper will appear in full in *Journal of the Optical Society*.

***7. Bergen Davis** — **Columbia University**

A RELATION BETWEEN THE CRITICAL POTENTIALS AND THE INDICES OF ELEMENTS AND COMPOUNDS

8. Paul D. Foote, T. Takamine and R. L. Chenault — **Bureau of Standards**

EXCITATION OF FORBIDDEN SPECTRAL LINES

While numerous examples of the breakdown of the selection principle for azimuthal quantum number occur in the spectroscopic literature, exceptions to the selection principle for inner quantum number are comparatively rare except in a strong magnetic field (second Paschen-Back effect).

We have found that the forbidden lines $1S-8p_1$ for Zn and Hg and $1S-2p_1$ and $1S-2p_2$ for Cd may be excited in the positive column of a low-voltage discharge through the heated vapor, using a tungsten hot cathode. Of these, only the classification $1S-2p_1$ for Hg has been noted previously. The lines $1S-2p_2$ for Zn and Hg lie in an unfavorable region so that definite conclusion in regard to their excitation cannot be drawn.

It is difficult to account for the phenomenon. An optimum current for a particular design of tube exists at which the effect is a maximum; with our tubes this current density was of the order 250 milliamperes/cm². If the excitation were due to the interatomic fields of neighboring atoms and ions, an incipient Stark effect, one should expect it to be accentuated by increasing the current to the joint where the sensitive pd series lines show broadening. The lines in question entirely disappear when other series lines exhibit this influence of interatomic fields.

Of course the strong excitation of $1S-2p_1$ and $1S-2p_2$ suggests a high concentration of atoms in the $2p_1$ and $2p_2$ states. K. T. Compton has shown that excited atoms are present in the positive column. In confirmation of this we have observed four sharp absorption lines λ2329, 2267, 2239 and 2306 in the positive glow of cadmium against the under-water spark or hydrogen tube as continuous background. These related lines are of the pp' type as is shown by the Zeeman effect for λ2306.

(Received January 21, 1925)

9. F. E. Wright — **Geophysical Laboratory, Washington, D. C.**

EMPIRICAL DISPERSION FORMULAE FOR OPTICAL GLASSES

A study, by graphical methods, of the relations between refractive indices and dispersions in the series of optical glasses shows that the following approximately straight line relations exist:

(1) $n_1-n_3=a\cdot(n_3-n_4)+b$, or $n_1=a\cdot(n_3-n_4)+n_3+b$;

(2) $\frac{n_1-1}{n_2-1}=c\cdot\frac{n_3-1}{n_3-1}+d$, or $n_1-1=c\cdot(n_3-1)+d\cdot(n_3-1)$; by combining two equations of this form we find the relation
$n=e\cdot(n_1-n_2)+f\cdot n_3+g$, in which $f+g=1$;

(3) $\frac{n_1-n_2}{n_3-n_4}=k\cdot\frac{n_5-n_6}{n_3-n_4}+m$, or $n_1-n_2=k\cdot(n_5-n_6)+m\cdot(n_3-n_4)$;

(4) $\frac{n_D-1}{n_F-n_C}=p\cdot\frac{n_1-n_2}{n_F-n_C}+q$;

(5) $n_1=h\cdot n_3+i$;

(6) $v_1=a\cdot v_3+b$.

Of these relations the first two are the most exact and from them the others can be derived; from them also empirical dispersion equations for optical glasses are most readily deduced. The dispersion equations which have been suggested and several new equations are discussed in the light of equations (1) and (2).

Bibliography:
T. Smith, Note on dispersion formulae and the secondary spectrum. Trans. Opt. Soc. *22*, 99-100, 1921; F. E. Wright, Dispersion in optical glasses, *Journal of the Optical Society of America 4*, 148-159, 195-204, 1920; *5*,389 397, 1921.

The present paper will appear in full in *Journal of the Optical Society of America*.

10. Irwin G. Priest **Bureau of Standards and Munsell Research Laboratory**

GRAY SKIES AND WHITE SNOW

At the Boston meeting of the Optical Society, October, 1924 (and previously) Mr. A. Ames, Jr., showed that reproduction, in a plane picture, of the accurate projection of an "objective scene" may result in an effect on the beholder of the picture which is neither natural nor artistic. Mr. Ames has shown that much more natural pictures, which recall the visual impression of the "objective scene" more pleasingly and vividly, may be made by introducing certain aberrations into the picture.

In the oral discussion of Mr. Ames' paper, I pointed out that a similar paradox may exist between the "values" (A. H. Munsell, "Color Notation," pp. 18 and 112) in a picture and the brightnesses in the "objective scene." My present purpose is to substantiate this proposition by specific experiments and quantitative photometric data.

The following major propositions will be demonstrated by experimental data.

(1) In viewing an actual scene of snow-covered ground and overcast sky, the sky may be perceived as "gray" and the snow as "white" even though the brightness (technical photometric sense) of the snow be much less than the brightness of the sky.

(2) The observer may be conscious that the sky is "brighter" than the snow in spite of the fact that he perceives it as "gray," and the snow as "white."

(3) This paradox cannot be wholly explained by assuming that the snow appears white because of contrast with neighboring dark objects.

(4) If a virtual image of the "gray" sky be brought into juxtaposition with the direct view of the "white" snow, the snow may appear "gray" and the image of the sky "white."

(5) A vertical wall illuminated by half of the sky on an overcast day may be perceived as "white" and the sky immediately above and beyond it as "gray" even though the sky be much brighter than the wall.

(6) Photographs which render the relative brightnesses of sky and white objects approximately as they would be measured photometrically may produce quite unnatural effects.

(7) To give a faithful rendition of the visual effect, the artist may be obliged to make the relative brightnesses on his canvas in reverse order of the corresponding brightnesses in the "objective scene."

Various illustrative and collateral data too extensive to abstract will be included in the paper.

* 11. **A. G. Shenstone** — **University of Toronto**

THE ARC SPECTRA OF SILVER AND COPPER

* 12. **A. G. Worthing** — **Nela Research Laboratory**

OPTICAL CONSTANTS OF TUNGSTEN AT INCANDESCENCE AND AT ROOM TEMPERATURE

* 13. **R. J. Piersol** — **Westinghouse Elec. & Mfg. Company**

RADIAL ASTIGMATISM IN LIGHT REFLECTED AT AN ANGLE FROM SPHERICAL MIRRORS

SATURDAY MORNING

Joint Session of The Optical Society of America and The American Physical Society

The Papers at this Session were contributed by The American Physical Society. For authors, titles, abstracts, etc., see program of that Society.

Besides these Papers, two Lectures were given at the invitation of The Optical Society of America, as follows:

Dr. C. E. Kenneth Mees — **Eastman Kodak Company**

THE PROPERTIES OF PHOTOGRAPHIC MATERIALS IN RELATION TO THEIR USE IN PHYSICAL MEASUREMENTS

The sensitive surface of photographic materials consists of a suspension of crystals of silver halide in gelatine. After development the silver halide is changed into silver and the amount of silver is measured by its optical density which for a considerable range of exposures is proportional to the logarithm of exposure given.

The measurement of the relation between density and exposure is what is termed "sensitometry," an important branch of which deals with the variation of sensitivity with wave length. Methods of photographic photometry depend either on the increase of the size of a small image with exposure or on the measurement of the increase of density.

The structure of the emulsion regulates the sharpness at the edge of an image, which is proportional to the contrast due to development and inversely proportional to the optical turbidity. The resolving power depends both on the sharpness and on the graininess of the emulsion and is measured experimentally.

Graininess is the subjective appearance of grain seen when an image is viewed under limited magnification.

The measurement of these various properties and their application in physical work will be discussed.

Bibliography:
Monographs on the Theory of Photography, "Physics of the Developed Photographic Image," V., by F. E. Ross
Monographs on the Theory of Photography, "Aerial Haze and its Effect of Photography from the Air," II

(Received January 21, 1925)

Dr. W. T. Bovie — **Harvard University**

THE EYE AS A LIGHT-SENSITIVE INSTRUMENT

SATURDAY AFTERNOON

Session of The American Physical Society

See program of that Society.

INSTRUMENT SECTION

A TORSION BALANCE FOR MEASURING RADIATION PRESSURE

By J. D. Tear

ABSTRACT

An apparatus for measuring radiation pressure is described which is suitable for demonstration purposes and which may prove to be of value for absolute radiation measurements. It consists of a silver vane torsion balance suspended in a metal case in an atmosphere of air or hydrogen.

Deflections at high vacuum were plotted against readings of a radiometer vacuum gauge. Extrapolaton to zero gas pressure gave a residual deflection corresponding to radiation pressure.

Deflections were independent of pressure, between 20 and 74 cm, for hydrogen and were equal to above residual deflection within the limits of experimental error, one-half per cent.

Errors due to gas forces were much larger for CO_2 than for air or hydrogen, and for hydrogen at atmospheric pressure were negligibly small.

The existence of a pressure due to reflection or absorption of radiation was demonstrated in 1901 by Nichols and Hull[1] at Dartmouth and by Lebedew[2] at the University of Moscow. Subsequent investigations by Nichols and Hull[3] established a quantitative confirmation of the Maxwell-Bartoli theory, within the limits of experimental error, approximately 2 per cent.

Repeated efforts to detect radiation pressure, dating from the time of Kepler, had failed because there were in all the experiments effects due to ununiformly heated gases, which by their magnitude masked the effect sought. Nichols and Hull used a torsion balance consisting of silvered glass vanes suspended in the axis of a large cylindrical glass jar. A relatively high pressure, about 16 mm of Hg, was found at which the gas effects were small, and a method was devised by which the residual disturbing forces could be corrected for. This type of pressure balance was later improved upon by Hull,[4] who, after a study of several different systems, concluded that, over a limited range of gas pressures, errors due to gas action could be reduced to less than 1 per cent.

[1] E. F. Nichols and G. F. Hull, Phys. Rev., *13*, p. 307, October 1901.
[2] Lebedew, Ann. Physik, *6*, p. 433, October 1901.
[3] Nichols and Hull, Phys. Rev., *17*, pp. 26 & 91, 1903; Ann. Physik, *12*, p. 255; 1903.
[4] G. F. Hull, Phys. Rev., *20*, p. 292, 1905.

Maxwell had suggested that the experiment be tried by allowing radiation to fall upon a thin metal vane suspended in a vacuum. This was the method adopted by Lebedew, though with the degree of vacuum attainable at that time there remained errors due to gas forces as large as 20 per cent. Because of the recent advances in high vacuum technique the method of eliminating spurious effects by evacuation is particularly attractive, and the recent experiments of Gerlach and Golsen[5] indicate that under favorable conditions it is indeed possible by prolonged evacuation to practically eliminate errors due to residual gases.

The requirements of a radiation pressure balance in vacuo are, however, very severe. It is necessary to maintain a pressure of 10^{-7} mm Hg or less for many hours, a condition difficult to fulfill if the system is such that it cannot be heated during evacuation. Moreover, in high vacuum a torsion balance is practically undamped and is extremely sensitive to vibrations. When once disturbed it is never observed to come to rest. Deflections are, therefore, measured by recording and averaging a large number of turning points, and it is only under the most favorable conditions that measurements can be made with any degree of precision.

An endeavor to obtain a radiation pressure apparatus suitable for a permanent exhibit, and which could be trusted to function without the attention of a skilled operator, led to a combination of the metal vane torsion balance as used by Lebedew and others with the high gas pressure feature of the Nichols and Hull experiment.* Additional developments, chiefly in the case and in the choice of the gas, resulted in so reducing errors due to gas effects that the instrument in its present form may prove of value for absolute measurements based upon the Maxwell-Bartoli theory of radiation pressure.

DESCRIPTION OF APPARATUS

The arrangement of apparatus is shown in Fig. 1A. Radiation from a coiled tungsten filament at F, limited by a 3 mm aperture in the diaphragm D, was brought to a focus upon one vane of the torsion balance by the concave mirror M. Rotation of the vane, following the opening or closing of the shutter T, was indicated by the deflection of a spot of light on the ground glass scale G. The image of F could be

[5] Gerlach and Golsen, Zs. f. Physik, *15*, p. 1-7, 1923; Golsen Ann. Physik, *73*, p. 624; 1924.

* An apparatus similar to that here described was built at the request of the late Dr. E. F., Nichols and presented by him to the National Academy of Science Museum, Washington, D. C.

moved from the left vane to the right vane and vice versa by means of a micrometer screw which bore against a steel point c at a known distance (28 cm) from the axis of rotation of M and which permitted this adjustment to be made within 0.001 mm. The total displacement of the image from left to right, 1.65 mm, could be repeated with as great an accuracy as the corresponding deflections could be read.

The suspended system, Fig. 1B, consisted of a plane, polished, silver vane ($2\times4.5\times0.015$ mm), and a concave platinum-on-glass mirror (1.5 mm dia.) mounted symmetrically upon a quartz rod (0.02 mm dia.) and suspended by a quartz fiber in the thick walled brass case as shown in Fig. 1C. The fiber was chosen by trial. Its torsion constant was such as to give the system a period of 12 sec. with the corresponding deflection time of 6 sec.

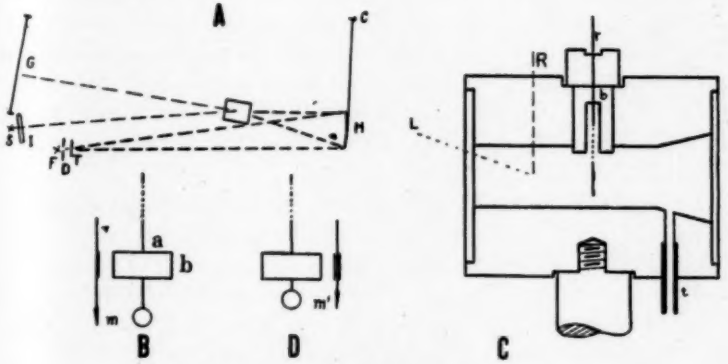

FIG. 1. A. *Arrangement of apparatus*
B. *Suspended system of torsion balance*
C. *Torsion balance case*
D. *Radiometer vacuum gauge*

Although the dimensions given are somewhat arbitrary there are certain general considerations which served as guides in the design of the balance. Nichols and Hull found that, in order to minimize the gas forces always present with this type of apparatus, the rise in temperature due to the incident radiation should be as small as possible and the system should be accurately symmetrical; i.e., the vanes should be plane, should hang vertically, and temperature differences between opposite vane faces should be small.

The first condition is favored by choosing for the vane a material of high reflecting power so that little of the incident energy is absorbed,

and also by attaining high sensitivity, thus insuring measurable deflections with incident radiation of low energy density. Referring to Fig. 1B, if the image of the source completely covers that portion of the vane to the right of the vertical axis, the radiation pressure p acting upon the area ab causes the system to rotate against the restoring force of the fiber, through a small angle θ. Neglecting the moment of inertia of the rod and mirror, elementary considerations give for the sensitivity of the system,

$$\frac{\theta}{p} = \frac{3P^2}{16\pi^2\rho t a}$$

where P is the free period, ρ the density of the material of the vane, and t the uniform vane thickness. In practise the period P is determined by the particular fiber chosen. There appears to be little advantage in so reducing the restoring force of the fiber as to give an inconveniently long period. Although the deflections may be magnified by so doing, their dependability is not necessarily increased. A real advantage is gained, however, by reducing the vane width, a. This advantage continues as long as the moment of inertia of the vane alone is greater than twice that of the remainder of the system.

Reduction in vane thickness, by tending toward greater sensitivity and by increasing heat conduction between opposite faces of the vane, favors both conditions mentioned above. It is possible by rolling annealed silver strips between polished steel plates to obtain highly polished vanes much thinner than the one here described. The necessity of planeness, however, has so far made it impractical to reduce the thickness below 0.01 mm. Silver is a particularly suitable material for the vane, both because it is a good reflector and because its ratio of heat conductivity to density is higher than that for other possible substances.

Although the above considerations are of importance in minimizing the errors introduced by the presence of gases, the successful functioning of the balance was found to depend, more than upon the design of the vanes, upon the nature of the gas and upon the metal case. When the system, Fig. 1B, was suspended in a glass bulb and exposed to radiation of known intensity the deflections observed (Fig. 2A) bore little relation to radiation pressure. They were usually negative at atmospheric pressure, changing to positive as the gas pressure within the bulb was reduced to the order of one millimeter, the region in which the "radiometer effect" begins to predominate. There was, of course,

an intermediate pressure at which the observed positive and negative gas forces equalled each other, and the torsion balance used at this pressure would have been free from gas effects. The necessity of determining this optimum gas pressure, and of maintaining it within certain narrow limits, was found, however, no longer to exist when the glass bulb was replaced by the heavy walled brass case. Though the pressure within the new case was varied from one to one-tenth atmosphere the deflection of the balance, for constant radiation, remained nearly constant and of the right magnitude to be attributable to radiation pressure.

This comparatively great freedom from the effects of gas forces would doubtless justify a series of experiments planned to determine the importance of the shape and material of the walls enclosing a system of this kind. The preliminary experiments indicated, however, that large thermal capacity, high thermal conductivity, and close proximity of the walls to the vanes were important requirements, and accordingly the case shown in Fig. 1C, with which the data here reported were taken, was chosen as embodying these features together with the advantage of simplicity of design.

The case was constructed of 2 in. (5.08 cm) drawn brass rod. A 15 mm axial hole was drilled and tapered as shown and closed at the ends by "plano" glass windows. The suspended system was lowered through the 1 cm hole into which the brass cylinder *b* was fitted, and was adjusted for height and zero position by means of the rod *r*. The vacuum seals were made with a mixture of beeswax and resin. Glass connections led from the tube, *t*, to the manometer and vacuum pump.

Errors due to gas forces. The characteristics of the gas forces involved are illustrated in the graphs in Fig. 2, which give torsion balance deflections for constant radiation as the gas pressure is varied. Curves B and B′ are for left and right vanes, respectively, of a typical system as shown in Fig. 1B in hydrogen and in the brass case described. In the region below 10^{-3} mm Hg the deflections approached a constant value, *p*, (Fig. 2) which, in accord with the investigations of Lebedew and with the recent and more accurate measurements of Gerlach and Golsen, may be assumed to correspond to radiation pressure. Between 0.01 and 0.1 mm the deflections became from four to five times as large as the residual deflection observed at very low pressures. The gas forces in this region were evidently due to the heating of the silver vane as a whole, as they produced positive deflections for B and negative deflections for B′, thus tending toward a rotation in the same sense,

whether the radiation fell upon the left or upon the right vane. This characteristic of the gas forces appeared with all the systems studied, some ten in all, and provided a means for partial cancellation of errors, the sum of left and right deflections being less in error than either deflection taken separately.

When the pressure became greater than 1 cm the deflections again approached the value p, the nearness of approach depending upon the construction of the vane and upon the nature of the gas in the metal case.

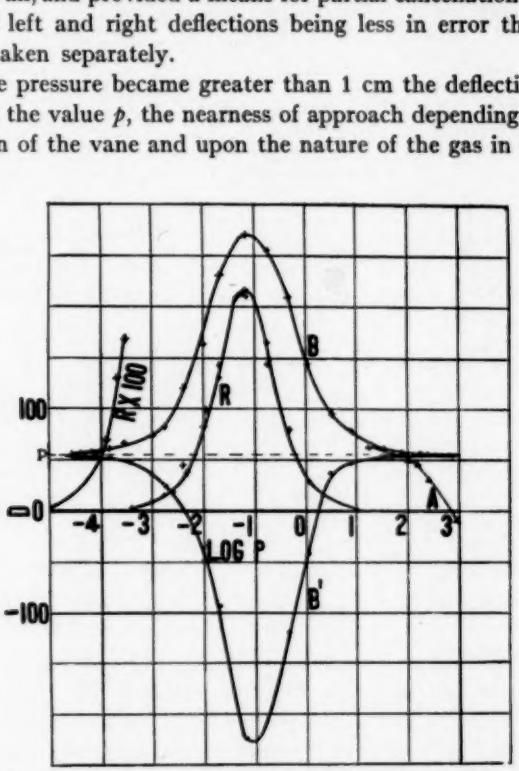

FIG. 2. *Curves showing the characteristics of the gas forces*
P, *gas pressure in mm of* Hg;
D, *deflections, mm, with scale at 75 cm.;*
A, *radiation pressure balance in air, in 10 cm glass bulb, left vane exposed to constant radiation;*
B, B′, *left and right vanes, respectively, of radiation pressure balance in brass case, in hydrogen;*
P, *deflection due to radiation pressure alone;*
R, *calibration curve of radiometer vacuum gauge.*

Left and right deflections for constant radiation, for atmospheres of hydrogen, air, and carbon dioxide, and for pressures between 30 and 745 mm, are given in Table 1. The deflections for the left vane show little variation for pressures above 10 cm or for the different gases.

The variations observed with the right vane are, however, very much larger, indicating a possible "warping" of this vane. Although the deflections vary as much as 30 per cent for the right vane as the pressure of the CO_2 changes from 176 to 745 mm, and 9 per cent for the same pressure change in air, the variations of the deflections in hydrogen within this pressure range are no greater than the expected errors in scale reading, about 0.2 mm. As the gas forces observed with the other gases varied markedly with pressure it is not likely that there remains a large constant error in hydrogen in this pressure range.

For pressures below 10 cm the deflections, in all cases, depart considerably from the constant value. The departures at all pressures are, however, such as to partically cancel when the left and right deflections are averaged.

The sums of left and right deflections for the three gases are represented graphically in Fig. 3a. The data for hydrogen are repeated in Fig. 3b with a different scale of ordinates.

Altogether five systems have been used in both air and hydrogen, two of which have also been used in carbon dioxide. In all cases the graphs of deflection against pressure were similar to those of Fig. 3a, though the magnitudes of the deflection variations were greater for some systems than for others. The maximum variations observed for air, between 10 cm and 75 cm, ranged from 10 per cent to 1 per cent. The variations for carbon dioxide for the two cases tried were respectively 5.6 and 3.7 times larger than the variations for air. In all cases the curves for carbon dioxide and air approached, for pressures between 10 and 20 cm, the constant value observed for hydrogen.

Relation between observed constant deflections and radiation pressure. It was pointed out with reference to Fig. 2, that upon continued evacuation the deflections approach a constant value which has been shown to be equal in magnitude to that predicted by the Maxwell-Bartoli theory of radiation pressure. The exact value of this residual deflection could be determined if one could measure the deflections over a wide range of low pressures and extrapolate the function so obtained to zero pressure. The inherent diffiiculties of measuring the actual pressures within a chamber of this kind, the walls of which are continually releasing a variety of gases and vapors in doubtless varying proportions, have been avoided by using as a pressure gauge a radiometer suspended in the chamber with the radiation pressure balance. The radiometer as used (see below) is insensitive to radiation pressure but extremely sensitive to the gas forces which introduce errors in radiation pressure

measurement. It is unnecessary then to know the gas pressure corresponding to a given radiometer deflection; it is only necessary to plot the radiation pressure balance deflections against radiometer deflections as the evacuation proceeds and to extrapolate the curve so obtained to zero radiometer deflection. If zero radiometer deflection corresponds to zero gas pressure the extrapolated value should correspond to true radiation pressure.

Radiometer Vacuum Gauge. The radiometer consisted of two mica vanes ($3\times7\times0.02$ mm) mounted on opposite sides of a 0.05 mm quartz rod as shown in Fig. 1D. The right half of the front vane and the left half of the rear vane were blackened with india ink. This system was suspended by a quartz fiber in the position indicated by the dotted line *R*, Fig. 1C, and mounted in the same manner as the radiation pressure balance, though the details of the mounting are omitted from the sketch. The period, with the particular fiber used, was approximately 2 sec. In position, the two mirrors *m* and *m'*, Figs. 1B and 1D, were at different levels, permitting both to cast images of the filament *S* upon the ground glass scale. A screen *I*, which transmitted only visible radiation, was placed in front of *S*. With this screen in place the stray light from this source produced no measurable deflection of either system. The radiometer was actuated by radiation from a 12-watt lamp *L*. The voltage of this lamp was controlled, thus making it possible to use the radiometer as a vacuum gauge over a wide range of pressures.

As the radiometer was symmetrical and uniformly illuminated it was compensated for radiation pressure. Had this compensation been in error by 5 per cent, the radiation pressure effect, computed from the constants of the system, for lamp *L* operated at the highest voltage used, would have been 0.5 mm on a scale at one meter. This error in the radiometer readings corresponds to an error of 0.1 per cent in the final radiation pressure measurements. As the compensation was probably much better than this, the radiation pressure effect has been neglected and the radiometer deflections considered due to gas forces only.

For purpose of illustration a calibration curve has been included in Fig. 2R, though accurate calibration is not important for the present experiment. For the extrapolation below 10^{-4} mm the deflections were assumed proportional to pressure, a relation found to hold quite strictly for pressures below 3×10^{-3} mm.

Measurements at High Vacuum. In Fig. 3, c and d, ordinates represent deflections of the radiation pressure balance, and abscissas radiometer deflections. One division on the horizontal scale corresponds roughly to a pressure of 10^{-5} mm of Hg.

In measuring left and right deflections (A and B, Fig. 3c) it was necessary to wait for the system to come to thermal equilibrium between observations of the deflections and the zero position, for, the radiation falling upon one vane and being partially absorbed established a lateral temperature gradient, and in addition elevated the temperature of the

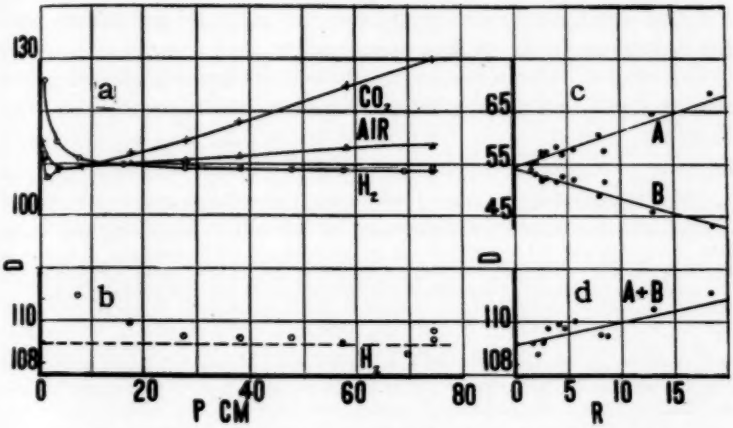

FIG. 3—*Radiation pressure balance in carbon dioxide, air, and hydrogen.*
D, *in c, left and right deflections individually elsewhere, the sum;*
A, B, *left and right deflections for high vacuum;*
R, *deflections of radiometer vacuum gauge—one division equals approximately* 10^{-5}*mm of mercury;*
Dotted line in b corresponds to the value of $A+B$ *when extrapolated to zero gas pressure.*

system as a whole. The gas effects due to the latter cause were eliminated in part by observing left and right deflections in immediate sequence, moving the image of the source from one vane to the other without closing the shutter. The sum of left and right deflections (A+B, Fig. 3d) could, therefore, be determined with greater consistency than either deflection taken separately. The apparently symmetrical distribution of the points in A and B, Fig. 3c, can be attributed to errors in determining zero position, as an error in the zero position causing the left deflection to be too small would cause the corresponding right deflection to be too large and vice versa.

Inasmuch as the gas forces which introduce errors in the radiation pressure measurements at high vacuum are presumably the same forces to which the radiometer deflections are due, for purposes of extrapolation the radiation pressure balance deflection has been assumed to be a linear function of the radiometer deflection. The least square straight lines, A and B, Fig. 3c, intersect the vertical axis at 54.5 and 54.3, respectively. The average deviation of the individual points from the lines is 0.8. The least square straight line A+B drawn through the sums of the left and right deflections intersects the vertical axis at 108.9, the average deviation of the points from the line being 0.4. In Fig. 3b a dotted line has been drawn through this point for comparison with the deflections for H_2 at high pressures.

Referring to Table 1, the mean deflections observed in air and carbon dioxide for the left vane, 54.1 and 54.5 are in good agreement with the value obtained above, 54.5, but those for the right vane are very much in error. From observations made with a number of systems, as previously mentioned, it seems likely that these large errors are due to some lack of symmetry accidentally introduced in mounting the vane.

TABLE 1. *Deflections of radiation pressure balance in hydrogen, air, and carbon dioxide at various gas pressures for constant radiation falling upon left and right vanes respectively*

H_2				AIR				CO_2			
Pmm	Left	Right	Sum	P	Left	Right	Sum	P	Left	Right	Sum
32	61.0	53.3	114.3	31	56.0	52.8	108.8	36	56.5	53.0	109.5
73	57.3	53.9	111.2	72	55.5	54.0	109.5	82	54.5	55.0	109.5
175	56.0	53.9	109.9	175	54.8	55.1	109.9	176	55.0	57.0	112.0
277	55.3	54.0	109.3	277	54.7	56.0	110.7	278	54.5	60.0	114.5
389	55.3	53.9	109.2	379	54.0	57.5	111.5	378	54.0	64.0	118.0
477	55.1	54.1	109.2					481	54.5	66.5	121.0
574	55.0	54.0	109.0	582	54.1	59.4	113.5	587	54.5	70.0	124.5
698	54.7	54.0	108.7								
745	55.6	53.8	109.4	745	53.2	60.3	113.5	745	54.5	75.0	129.5
Mean	55.3	54.0	109.2		54.1				54.5		

The left and right deflections in hydrogen show little variation with pressure for pressures above 20 cm., and both agree with the values obtained at high vacuum within the limits of error of the latter measurements.

The mean value of the sum of left and right deflections in hydrogen for pressures between 176 and 745 mm., 109.2 ± 0.2, is equal to the residual deflection observed at high vacuum, 108.9 ± 0.4 within the limits

of error, and in all probability corresponds, within these limits, to true radiation pressure.

CONCLUSIONS

Radiation pressure measurements may be made with a torsion balance, as here described, operated at atmospheric pressure. The errors due to gas forces are less for hydrogen than for air or carbon dioxide, and for hydrogen can be made less than one-half per cent.

NELA RESEARCH LABORATORY,
DECEMBER 13, 1924.

Soft X-rays from Iron.—Usual method of bombarding a metal sheet with electrons from a filament heated by a current i_0, varying the bombarding voltage V, measuring the photoelectric current i from a platinum plate facing the sheet, plotting i/i_0 versus V and looking for "breaks" in the curves. Photoelectric current was observed at values of V so low as 2.3 volts (but nothing is said about possible corrections for contact P.D.). This value is unprecedentedly low; possibly this occurred because the bombarding current was unprecedently high, an idea which is suggested by the fact that the filament was hot enough to produce a photoelectric current from the collector by virtue of its own radiation. Furthermore, quanta of energy as low as 2.3 volts should not be able to elicit electrons from a metal with so high a threshold-frequency as platinum. To account for this, Thomas suggests that the electrons striking the target excite radiation only after their initial energy has been supplemented by the energy acquired in falling inward (or "backwards") through the surface-potential-drop; a rather interesting idea. The curves, which are extended up as far as 750 volts, show numerous breaks, but they are nearly all of the inconspicuous sort. However, the author remarks that they are very accurately reproducible, and some of them agree with breaks earlier found by other observers.—[C. H. Thomas, Princeton; Phys. Rev. *25*, pp. 322-332, 1925.]

KARL K. DARROW

The L_3 Absorption-Frequencies of the Elements of the second Row.—The element is examined in the form of a gaseous hydride (SiH_4, PH_3, SH_2, HCl) except argon which is examined directly. The ionization current produced by a beam of heterogeneous x-rays, entering the gas through a thin celluloid film, is plotted versus the voltage of the electrons producing the x-rays; a break in the curve indicates the critical absorption-frequency. These lie closely upon a smooth parabolic cu ve, and so also does the value for Al, determined by finding a break in the curve representing the transmission of the x-rays through a sheet of Al as function of the electron-driving voltage. Comparisons with data by other observers show agreement with some and sharp discord with others.—[F. Holweck, Institut du Radium, Paris; C. R. *180*, pp. 658-662; 1925.]

KARL K. DARROW

Absorption of Light by excited Neon Atoms.—The normal level of the valence-electron of the neon atom lies so far below the next adjacent level that light of the visible spectrum is not of high enough frequency to lift it out; the emission-lines in the visible spectrum of neon are emitted by electrons moving from one to another of the outer levels, and cannot be absorbed except by a neon atom of which the electron is already displaced into an outer level. When light from a neon discharge tube passes through another such tube, the emission lines are not weakened until a discharge is started in the second tube, when some are much weakened, others comparatively little, others not at all. A photograph shows this quite clearly. The ones which are most weakened are those which can be absorbed by an atom having its valence-electron in the so-called s_3 level, which indicates that this level is the one into which the valence-electron is most frequently driven by the agencies acting in a discharge, or else the one in which it sojourns longest when once established, or perhaps both. An experiment in which the two tubes were excited by alternate half-cycles of a 100-cycle wave showed that there was no absorption, i.e., that the neon atoms do not remain in any of these excited states as much as 0.01 second. Possibly the duration of the excited state could be determined by such experiments with higher frequency.—[K. W. Meissner, Zürich; Ann. d. Phys. *76*, pp. 124-144, 1924.]

KARL K. DARROW

Absolute measurements on absorption of X-rays in Metals.—These are made by a balance method copied after Siegbahn, by which the effect of fluctuations in intensity in the x-ray beam is obviated. The beam is divided into two by appropriately-placed slits, and these are diffracted from one and the same crystal-face into similar ionization chambers; in the path of one beam, a wedge of aluminium is placed, in the path of the other a rotating sector, which diminishes by 1/2 the intensity of the latter beam; the thickness of aluminium required to reduce the intensity of the former beam in the same proportion is determined by interpolation among readings taken with various thicknesses (i.e. various settings of the wedge). Measurements on foils of Al and of other metals are made by means of the thus-calibrated wedge. The wave lengths used range from 0.3 to 0.81A. The data for aluminium, corrected for the iron the foils contain, agree admirably with Richtmyer's. The K absorption edge is well exhibited for the metals Cu, Ag, Sn and Pd, and the L absorption edges very well exhibited for uranium. The increase in absorption due to the L_3 electrons is equal to that due to the L_1 and L_2 electrons together.—[E. C. Stoner and L. H. Martin, Cavendish; Proc. Roy. Soc. *A107*, pp. 312-340, 1925.]

KARL K. DARROW

A DIRECT SYSTEM OF DESIGN FOR THE CEMENTED TWO LENS TELESCOPE OBJECTIVE

WITH SPECIAL REFERENCE TO THE COMA-FREE TYPE

By G. W. Moffitt

INTRODUCTION

The algebraic equations for the spherical aberration and for the coma of systems of "thin" lenses are, of course, well known to all who have made a study of optical design. It is also well known that the introduction of suitable practical thicknesses into such lens systems without changing the radii of curvature does not materially change the spherical aberration and the coma. On the other hand the addition of thicknesses in this manner *does* change the chromatic aberration considerably. In order to regain the chromatic balance the curves must be changed. These changes will, in general, introduce spherical aberration and coma.

Designers differ in opinion as to the best procedure to be followed from this point forward in the practical task of evolving the formula for an ordinary telescope objective of two lenses cemented together. Some rely entirely on trial and error methods to bring the residual aberrations to the desired small values simultaneously. Others trace rays through the tentative system and then, by the use of correction formulas, determine the changes to make in the radii of curvature in order that the aberrations may be simultaneously corrected. Still others, while using these tentative methods, are of the opinion that more direct methods of attack ought to be developed and made known which, while based on empiricism, furnish the practical designer with a straightforward and abbreviated course in the production of a formula.

The studies to be reported in this paper have been made in accordance with the last mentioned point of view. It is hoped, however, that the designer may find some bit of information or other that will be of use to him regardless of his views and methods. To the student of geometrica' optics it may furnish another vantage point from which to view the subject. For the novice 'n practical design it may serve as a simple and straightforward system with which he can get results even before he has made a comprehensive study of the theory of objectives. The method reduces the amount of computing required and, at the same time, the designer may ascertain before commencing the calculations

just what kind of a lens the glasses to be used will give. He is thus able to select the most suitable glasses without preliminary calculations, or at least with very short and simple ones.

It should be remembered, however, that the paper deals with only one of the many problems of optical designing—and a comparatively simple problem it is too. Moreover, it deals only with the adjustment of the aberrations to small values and does not deal at all with that more involved problem of determining the proper values for the residual aberrations in order that a good balance of the compensation of one part of an instrument with the others may be made. There is no substitute for experience in this matter of the fine adjustment of compensation.

The excellent and valuable work of Harting[1] constitutes the nearest approach to the desired material that seems to have been published heretofore. His method is still an inverse one to a certain degree, however. In the present paper this inverseness has been completely removed, at least as far as the user of the results is concerned, and it will be obvious after a cursory study that the point of view and the actual procedure are very different from those of Harting's method. A study of the data in Harting's tables, rearranged in various ways, leads one to suspect the existence of certain linear relationships that would be useful to the designer. Detailed empirical study of these questions disclosed the fact that some of these suspected relationships were linear to a surprisingly precise degree, while others did not stand up at all well and had to be abandoned.

NOTATION AND SIGN CONVENTION

Because of the conflicting systems of notation and sign convention that have appeared in the writings on geometrical optics, it is desirable to state the particular system used in a discussion. In this paper the following conventions regarding sign will be observed. The symbols to be used will be self-explanatory, or their significance will be apparent from the context.

Radii of Curvature.—A radius of curvature is to be given the positive sign if the light reaches the surface before it has passed the center of curvature of the surface. This is the system used by the majority of designers, and lens specifications are usually written in accordance with it. In conformity with what seems to be customary practice the light

[1] H. Harting, Zeitschr. f. Instrumentenkunde, *18*, pp. 357-380, 1898.

will be assumed to travel from left to right through the system wherever illustrated.

Thicknesses and Separations.—These will be regarded as intrinsically positive in all cases.

Angles.—The angles that the designer is concerned with are the angles that a ray makes with the geometrical axis of the system—frequently called slope angles—and the angles of incidence and refraction as ordinarily defined.

The slope angles will be given the positive sign if a line must be rotated less than 90° in an anticlockwise direction in order to pass from the geometrical axis of the system to the ray in question. This is in accordance with the conventions adopted in polar coordinate geometry.

Angles of incidence and refraction will be given the positive sign if a line must be rotated less than 90° in an anticlockwise direction in order to pass from the normal to the surface at the point of incidence to the ray in question. This also is in accordance with the conventions of polar coordinate geometry.

Chromatic Aberration.—This is to be given the negative sign if the longitudinal chromatic aberration for the two colors in question is of the kind found in an uncorrected single lens having the same general optical effect as far as Gaussian imagery is concerned. Thus the chromatically overcorrected lens is to be regarded as having positive chromatic aberration.

Spherical Aberration.—This is to be given the negative sign if the residual of longitudinal aberration is of the same kind as is present in a single lens of the same general characteristics with respect to Gaussian imagery. That is, the longitudinal spherical aberration of an uncorrected converging lens is to be given the negative sign.

Sine Condition Error.—This is to be given the positive sign if the numerical magnitude of $\frac{h}{\sin V}$ minus the numerical magnitude of the axial equivalent focal length comes out positive; negative if the difference comes out negative.

Coma.—This is to be given the positive sign if the algebraic difference, *Sine condition minus Spherical Aberration*, comes out positive. Negative if the opposite is the case. With this notation positive coma means outward coma, that is, coma streamers pointing away from the axis of the lens, or magnification too great for the outer zones of the system.

THE ACHROMATIC SOLUTION

Let it be assumed that the optical properties of a pair of glasses to be used in a cemented two lens achromat of the crown leading type are

known, and that the thicknesses, t_{cr} and t_{fl}, and the cemented radius of curvature, r_2, have been selected. It remains to determine the values of r_1 and r_3 that will give a stated equivalent focal length and freedom from chromatic aberration for two selected spectral lines. Tables might have been prepared, but since these radii depend to some extent on the thicknesses selected it seemed doubtful whether the preparation of tables sufficiently comprehensive was worth while. For the purposes of this paper at least it seemed best to give a formula used by the author for some time past in the direct solution of this specific problem. The method possesses a degree of logical directness and practical simplicity that may appeal to those who object to the use of trial and error methods. Any skilled computer who has taken the trouble to master the routine will be able to make the solution in half an hour or less if he has a calculating machine at all suited to the work.

Derivation of the Formulas.—There are, of course, many ways of deriving the necessary formulas, but perhaps it will be well to consider the problem as analogous to that of a system of three simple lenses separated by air spaces in which

$$\frac{1}{f_1}=\frac{n_1-1}{r_1}\;;\quad \frac{1}{f_2}=\frac{n_2-n_1}{r_2}\;;\quad \frac{1}{f_3}=\frac{1-n_2}{r_3}$$

$$S_1=\frac{t_1}{n_1}\quad ;\quad S_2=\frac{t_2}{n_2} \tag{1}$$

By the application of the well known formulas for the equivalent focal length of an air spaced system the reciprocal of the equivalent focal length of the combination is found to be

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}-\frac{S_1}{f_1f_2}-\frac{S_1}{f_1f_3}-\frac{S_2}{f_2f_3}-\frac{S_2}{f_1f_3}+\frac{S_1S_2}{f_1f_2f_3}\,. \tag{2}$$

Collecting terms and transposing,

$$\left(\frac{S_1S_2}{f_2}-S_1-S_2\right)\frac{1}{f_1f_3}+\left(1-\frac{S_2}{f_2}\right)\frac{1}{f_3}=\left(\frac{1}{F}-\frac{1}{f_2}\right)-\left(1-\frac{S_1}{f_2}\right)\frac{1}{f_1}\,. \tag{3}$$

Introducing the values of the f's and setting $1/r=\rho$ gives

$$\left[\left(\frac{S_1S_2}{f_2}-S_1-S_2\right)(n_1-1)\,(1-n_2)\right]\rho_1\rho_2+\left[\left(1-\frac{S_2}{f_2}\right)(1-n_2)\right]\rho_2 \tag{4}$$

$$=\left(\frac{1}{F}-\frac{1}{f_2}\right)-\left[\left(1-\frac{S_1}{f_2}\right)(n_1-1)\right]\rho_1\,.$$

which is of the form

$$C\rho_1\rho_3+B\rho_3=D-A\rho_1 \tag{5}$$

Solving for ρ_3;

$$\rho_3=\frac{D-A\rho_1}{C\rho_1+B}\cdot \tag{6}$$

For achromatism for two colors, say for the C and the F lines,

$$\frac{D_C-A_C\rho_1}{C_C\rho_1+B_C}=\frac{D_F-A_F\rho_1}{C_F\rho_1+B_F}=\rho_3\,. \tag{7}$$

Clearing of fractions and collecting terms,

$$(A_CC_F-A_FC_C)\rho_1^2+(A_CB_F-A_FB_C-C_FD_C+C_CD_F)\rho_1 +(B_CD_F-B_FD_C)=0\,. \tag{8}$$

which is an ordinary quadratic in ρ_1. A determination of the numerical coefficients of this equation requires a calculation of the values of A, B, C and D for the two colors, that is, eight constants in all. As will be seen from Equation (4), these are functions of the glass data, the cemented radius of curvature and the thicknesses, as follows:

$$A=(n_1-1)\;-\;\left(t_1-\frac{t_1}{n_1}\right)\left(\frac{n_2-n_1}{r_2}\right)\cdot \tag{9}$$

$$B=-(n_2-1)+\left(t_2-\frac{t_2}{n_2}\right)\;\left(\frac{n_2-n_1}{r_2}\right)\cdot \tag{10}$$

$$C=\left(\frac{t_1}{n_1}\cdot\frac{t_2}{n_2}\cdot\frac{n_2-n_1}{r_2}-\frac{t_1}{n_1}-\frac{t_2}{n_2}\right)(n_1-1)\,(1-n_2)\cdot \tag{11}$$

$$D=\frac{1}{F}-\left(\frac{n_2-n_1}{r_2}\right)\cdot \tag{12}$$

The coefficients of the quadratic may then be found by properly combining these constants according to Equation (8), after which the equation may be solved for ρ_1 or for r_1. This work is best done on a machine of suitable design.

The third radius, r_3, may then be found by substitution in Equation (6), which however may well be converted into the following form for the purpose.

$$r_3=\frac{C+Br_1}{Dr_1-A}\cdot \tag{13}$$

The constants for either the C or the F line may be used, or better still, both may be used and two results obtained thus serving as a kind of check.

THE FORM CHARACTERISTICS

Consider the series of crown leading achromats obtained with a given pair of glasses by holding the thicknesses constant and varying the form in such a way that the equivalent focal length and the achromatism remain constant. In such a series the spherical aberration and the coma will vary continuously in some manner or other as this form change proceeds. An empirical investigation would enable one to plot these residual aberrations precisely as functions of any convenient variable that definitely fixes the form of the lens according to the conditions stated above. It has been found convenient to use $1/r_2$, usually designated by ρ_2, as the independent variable and to plot only its numerical magnitude.

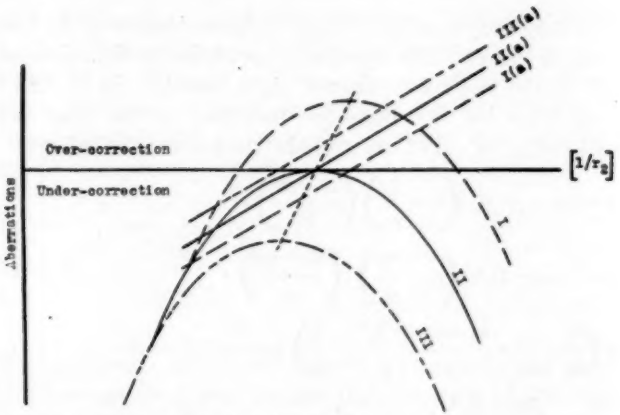

Fig. 1. *Form characteristics of spherical aberration and coma.*
I & Ia *for large value of* $n_g - n_{e'}$.
II & IIa *for a glass pair capable of forming a coma-free cemented objective.*
III & IIIa *for a single lens.*

The form characteristic curve of spherical aberration thus obtained is essentially a parabola with its axis parallel to the axis of aberrations and with its branches opening outward toward the region of negative aberration (undercorrection). See Fig. 1. The form characteristic curve of coma is practically a straight line in the region of useful lenses. A vertical line through the point of zero coma will always cut the spherical aberration curve at, or very near, its maximum. Thus the coma-free objective of the cemented type is always of the form that gives the maximum tendency toward overcorrection of spherical aberration; that is, the peak of the parabola. Curve I, Fig. 1, is for a system of the

crown leading type in which there is a large difference between the mean indices of the two glasses. Considerable overcorrection of spherical aberration is possible in this case. There are two forms of the objective for which the spherical aberration is zero; the one having positive coma and the other negative coma of about the same magnitude. The parabola moves downward on the diagram as the difference of the mean indices decreases, and when this difference becomes zero the parabola lies entirely below the horizontal axis in a position corresponding to that of a single lens. See curve III in Fig. 1. When the peak of the parabola lies on the horizontal axis (line of zero aberration) there is only one form of the lens that annuls the spherical aberration. Since the value of $1/r_2$ that corresponds to the peak of the spherical aberration curve is the one that results in zero coma, it is obvious that this form is also free from coma. Thus it is seen that in order to obtain a cemented objective that shall be free from spherical aberration and coma simultaneously, it is necessary to choose the glasses so that the peak of the spherical curve just reaches the line of zero aberrations—or perhaps, for practical reasons, comes very slightly above it—and then to choose the form that gives zero coma. Such an objective cannot be overcorrected for spherical aberration. Curve II, Fig. 1.

It is interesting to note, moreover, that in this region small changes in form produce only slight effects on the value of the residual spherical aberration, since the curve is practically horizontal at this point. But since the coma curve is a straight line there is no preferred form or combination of glasses as far as the reduction of the coma effect of form variation is concerned. In fact, the proportionality factor between form change (ρ_2 change) and the resulting coma change is surprisingly constant throughout the entire domain of available glasses.

Obviously combinations that lead to spherical aberration curves lying wholly in the region of undercorrection are of little interest to the designer of telescope objectives. Those giving a parabola extending into the region of overcorrection are of interest in the design of objectives in which residual coma is desired. But those combinations leading to a parabola with a peak very close to the line of zero aberrations are of special interest because they can be adjusted to give achromatic objectives that are simultaneously corrected for spherical aberration and coma,—lenses that are particularly useful in many instruments.

It is thus seen that a knowledge of the factors that influence the position of the form characteristic curves of spherical aberration and of coma, is very useful to the designer. Whenever the residual spherical

aberration and coma are stated for any given combination, it is possible to say at once in which direction to change the form in order to come nearer a desired adjustment. In fact, since the slope of the coma curve is a constant of known value it is usually possible to say *how much* to change the form in order to bring about the desired result. For example, if the spherical aberration were +0.5 and the coma +0.4 the system could not be simultaneously corrected for these two aberrations since it is already showing overcorrection for spherical aberration. Moreover, since the slope of the coma curve is always practically 40, a decrease of 0.01 in the magnitude of ρ_2 would be necessary to free the system from coma, although if this change were made spherical aberration would be still farther overcorrected.

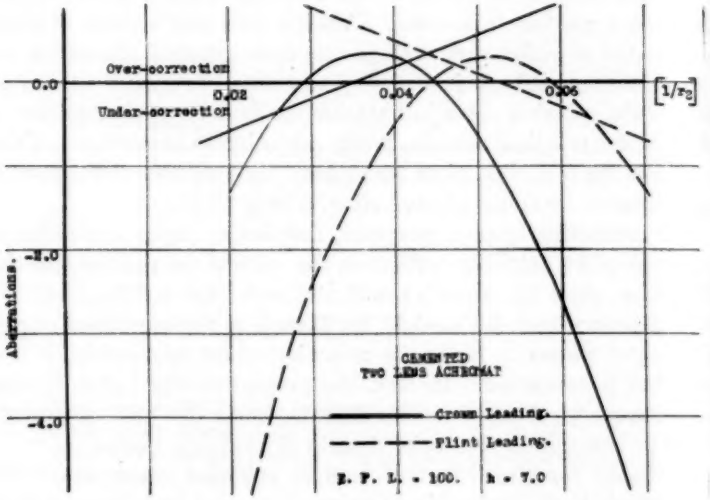

FIG. 2

Curves similar to those already described may be obtained for the flint leading type of objective, although they lie farther to the right on the chart (larger values of ρ_2) and the slope of the coma line is of the opposite (negative) sign, provided it be understood throughout that *magnitudes only* of ρ_2 be used as abscissae. The peaks of the parabolas are very nearly at the same height in both cases.

In Fig. 2 the characteristic curves for a typical pair of glasses are plotted in order to give an idea of the magnitude of the quantities dealt with in a practical case. It happens that this particular combination

leads to an interesting special result. One of the crown leading forms giving zero spherical aberration is practically the same as one of the flint leading forms giving the same result. Both have practically the same value of positive coma. In other words, with this pair of glasses there is a certain form of objective spherically corrected and having the same residual coma with *either* side toward the object.

The discussion will now be confined to the crown leading type, as it calls for the more moderate curves and seems to be generally favored. What has been done for this type could also be done for the flint leading type if desired.

THE CROWN LEADING COMA-FREE TYPE

As has been pointed out above, a corrected cemented objective of this particularly useful and desirable type can be obtained if the glasses are so selected that the spherical aberration maximum falls on the line of zero aberration or slightly above it. In order to definitely determine the relations that must exist among the constants of the glasses when this condition prevails, a number of lenses were carefully computed at strategic points distributed throughout a glass domain the boundaries of which were somewhat more extended than necessary to include existing optical glasses suitable for practical purposes.

Throughout the work an equivalent focal length of 100 was maintained for the *C* and *F* lines, while the ray tracing was done on the *D* line for a ray entering the system at a height of 7.0 from the axis. The thickness of the crown was held at 3.5 and that of the flint at 1.75. Spherical aberration and coma were adjusted to zero in each case by changing the mean index of one of the glasses and the form of the objective until the desired result was obtained.

Arbitrarily chosen values of the Δn ratio,—defined as Δn ratio $= \dfrac{(n_F - n_C)_{Fl}}{(n_F - n_C)_{Cr}} = \dfrac{\Delta n_{Fl}}{\Delta n_{Cr}}$,— were used in order to systematize the work still more. Data were collected on series of lenses having Δn ratios of 1.4, 1.6, 1.8, 2.0, 2.2 and 2.4. From these any intermediate value can be determined quite accurately. The results are shown graphically in Fig. 3, in which $(n_D - 1)_{Cr}$ is plotted against $(n_{\bar{D}} - 1)_{Fl}$ for the various values of the Δn ratio. Even though the work was carried to the fourth decimal place in the indices the curves are sensibly straight lines. They are thus more than amply precise for the ordinary needs of practical design. Moreover, the lines seem to have a common point of intersection. To investigate this question the slopes of the lines were plotted

against the $(n_D-1)_{Cr}$ intercepts and a straight line was found to fit as well as any other curve. The line having a Δn ratio of 2.4 is the only one that does not seem to be in harmony with the others in this respect.

A solution for the best values for the coordinates of the common point of intersection gave (.17007, .17230). On the assumption that all the lines pass through this point the best values for the slopes of the several lines are shown in Table 1. Linear interpolation will suffice for

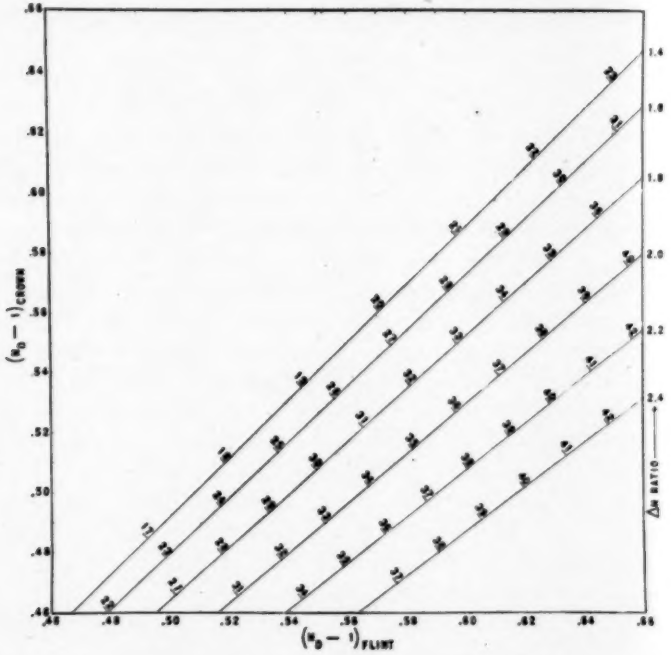

Fig. 3. *Showing the relations that must exist among the optical constants of a pair of glasses if they are to combine in a corrected crown-leading objective free from coma. The numbers on the Δn lines are approximate values of the second radius of curvature.*

the determination of intermediate values. The figures are sufficiently accurate to lead to determinations of $(n_D-1)_{Fl}$ that match stated values of $(n_D-1)_{Cr}$ and Δn ratio with an uncertainty of about one unit in the fourth decimal place except for the region near the ends of the 2.4 line where the error may reach 0.0009 at the extreme ends of that portion of the line shown in Fig. 3.

The striking simplicity of the glass relationships, when expressed in this way, makes it a very easy matter to ascertain whether a proposed pair of glasses can be combined to form a cemented objective of the coma-free type by simply consulting the chart shown in Fig. 3. If the chart be drawn carefully on a large scale there need be no calculation whatever—unless it be for other adjustment of the residual aberrations or for other relative apertures. The data of Table 1 suffices for the construction of such a chart. Or, if one prefers, the simple linear calculation may be made each time and the use of the chart dispensed with.

TABLE 1. *On the assumption that all of the lines intersect at* (0.17007, 0.17230), *the best slope values are as follows*

Δn Ratio	Slope	Δn Ratio	Slope	Δn Ratio	Slope
1.40	0.96967	1.75	0.8967_5	2.10	0.8051
1.45	0.9614_5	1.80	0.88436	2.15	0.7934
1.50	0.9523	1.85	0.8715_5	2.20	0.78058
1.55	0.9424_5	1.90	0.8585_5	2.25	0.7684
1.60	0.93181	1.95	0.8452_5	2.30	0.7561
1.65	0.9205	2.00	0.83195	2.35	0.7440_5
1.70	0.9088	2.05	0.8189_5	2.40	0.73201

The Selection of Glass for the Coma-Free Objective.—The next step in the general problem of the coma-free objective—and the one the designer is most likely to encounter first—is the selection of, say a flint glass to go with a specified crown glass. To facilitate the answering of this question the method illustrated in Fig. 4 may be employed, using either graphical charts carefully drawn, or performing the simple calculations and finally drawing the resulting line on the glass chart. In the figure the first quadrant represents sketchily a large scale reproduction of Fig. 3. In the second quadrant, and again in the fourth, (n_D-1) is plotted against Δn for all known glasses within the boundaries of the chart. The third quadrant serves merely for finding graphically the value of Δn_{Fl} that goes with a given value of Δn_{Cr} to give a desired Δn ratio. The second quadrant may thus be looked upon as the domain of glasses to be used in the positive or crown component, while the fourth quadrant serves in like manner for the negative, or flint, component.

To ascertain which flints, if any, will combine with a given crown glass proceed as follows. From the point in the crown glass quadrant that represents the given crown glass proceed along the horizontal into the first quadrant until a convenient Δn ratio line is reached, for

example, the 1.4 line. Drop a vertical line from this point into the fourth quadrant. There is one point on this line that represents a suitable glass. To find it go back to the starting point in the second quadrant, and drop a vertical line from it into the third quadrant. At the point where this vertical intersects the Δn ratio line in the third quadrant draw a horizontal into the fourth quadrant until it intersects

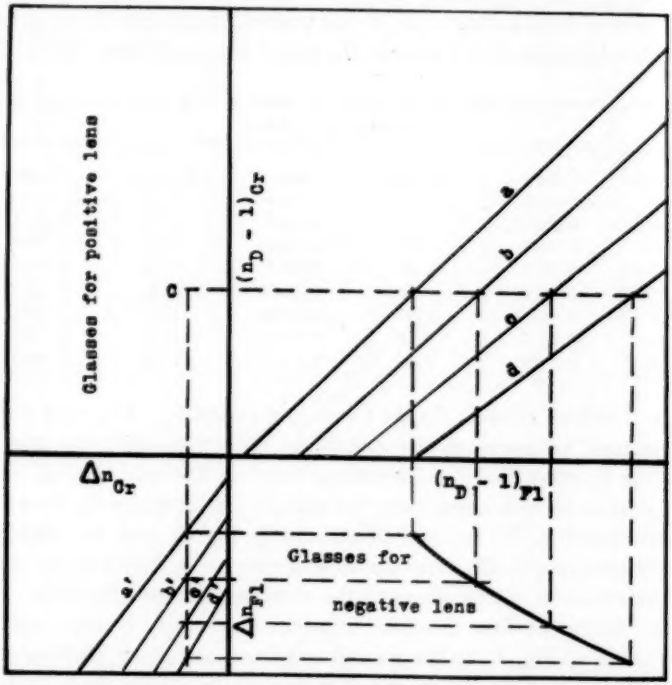

Fig. 4. *Schematic representation of the method for finding the flint glasses that will combine with a given crown, to form coma free corrected objectives of the crown leading type.*

the vertical line already drawn. This intersection represents the suitable flint glass. It will be noticed, however, that the Δn ratio was arbitrarily chosen. In order to find all the glasses that will combine with the crown in question repeat the process several times with widely different Δn ratios. Then draw a curve through the points in the fourth quadrant thus found. This line is the locus of points representing the glasses suitable for combining with the specified crown to form objectives of the coma-free type. If any real glasses fall on or near it the

designer may select that one which seems to be best suited to his purposes.

The Selection of the Cemented Radius.—By means of the foregoing charts and tables, or of the equivalent simple calculations, a proper choice of glasses may be made before any of the actual work of designing has been started. The designer can then proceed with full assurance that when he has selected the proper radii of curvature, the residual aberrations will be very close to the intended small values. The determination of the proper radii of curvature is then the next step.

For this purpose one might employ the algebraic method applicable to the "thin" objective (as in H. D. Taylor's System of Applied Optics, for example) and so ascertain the approximate radii that would eliminate either spherical aberration or coma. Since the coma-free objective is being considered either one will do. One would use only the value for r_2 so obtained however. This would be inserted in the achromatic solution already described, so that the proper values of r_1 and r_3 for the thicknesses chosen would be obtained.

In many cases the objective resulting from the above procedure would not be adjusted precisely enough. Moreover, the average designer does not take kindly to the particular type of algebraic exercise required. On the other hand, it has been possible to provide a good table of the proper values of r_2 in connection with the work done to establish the position of the lines on the glass charts. By supplementing these data with another group intended for checking purposes, it has been made possible to compile Table 2, in which precise values of r_2 will be found for almost any possible combination. The values given are for an equivalent focal length of 100 for C and F and for the thicknesses used throughout this discussion, namely, 3.5 for the crown and 1.75 for the flint. Linear interpolation in both directions will suffice for values between those given in the table. The tabular values are accurate within a very few units in the second decimal place. This is a degree of precision greater than is often needed. In fact, for most ordinary work the second decimal place may be disregarded entirely. To that degree of precision r_2 becomes a linear function of either (n_D-1) for any stated Δn ratio.

With r_2 thus easily determined, one may proceed with the achromatic solution for r_1 and r_3 as already described. If the thicknesses are to be different from those used in this discussion they should be used in the solution. In such cases slight corrections may be needed in the tabular value of r_2 although such corrections would be quite small and usually

TABLE 2. *Values of r_2 for Coma-free Combinations*

Crown n_D	Δn ratio										
	1.40	1.50	1.60	1.70	1.80	1.90	2.00	2.10	2.20	2.30	2.40
1.46	−16.04	−19.24	−21.99	−24.45	−26.70	−28.70	−30.55	−32.15	−33.60	−34.95	−36.00
.47	−16.35	−19.65	−22.51	−25.04	−27.38	−29.44	−31.32	−32.97	−34.43	−35.82	−36.95
.48	−16.70	−20.08	−23.04	−25.64	−28.06	−30.17	−32.10	−33.78	−35.26	−36.69	−37.90
.49	−17.06	−20.53	−23.57	−26.25	−28.74	−30.90	−32.87	−34.59	−36.10	−37.56	−38.85
1.50	−17.43	−20.96	−24.10	−26.87	−29.44	−31.66	−33.67	−35.42	−36.95	−38.44	−39.80
.51	−17.81	−21.43	−24.64	−27.50	−30.13	−32.41	−34.47	−36.24	−37.80	−39.32	−40.75
.52	−18.20	−21.89	−25.19	−28.12	−30.83	−33.18	−35.29	−37.10	−38.66	−40.19	−41.71
.53	−18.60	−22.36	−25.74	−28.77	−31.53	−33.95	−36.11	−37.93	−39.51	−41.08	−42.67
.54	−19.00	−22.84	−26.29	−29.41	−32.23	−34.73	−36.94	−38.77	−40.37	−41.97	
.55	−19.40	−23.30	−26.85	−30.09	−32.93	−35.51	−37.78	−39.62	−41.23		
.56	−19.80	−23.79	−27.42	−30.69	−33.65	−36.31	−38.62	−40.47	−42.10		
.57	−20.21	−24.28	−27.99	−31.34	−34.37	−37.10	−39.47	−41.33			
.58	−20.62	−24.76	−28.57	−31.99	−35.08	−37.88	−40.32				
.59	−21.03	−25.26	−29.15	−32.65	−35.80	−38.66					
1.60	−21.44	−25.76	−29.74	−33.31	−36.52	−39.45					
.61	−21.85	−26.24	−30.33	−33.98	−37.26						
.62	−22.26	−26.75	−30.92	−34.63							
.63	−22.68	−27.24	−31.51								
.64	−23.10	−27.74									
.65	−23.52										

negligible for the thickness changes that would occur in practical design.

In the majority of cases the dimensions found by the procedure outlined above will be those of the finished lens. One would seldom rely on an unchecked piece of work, however, and therefore it is good practice to calculate the usual properties of the system for the purpose of checking the solution and of providing a basis on which to work in case slight readjustments should prove desirable.

THICKNESS EFFECTS

The standard thicknesses adopted in this discussion cannot always be used. Therefore it becomes necessary to know what the effect of any reasonable departure from these standard thicknesses will be on spherical aberration and on coma, assuming of course, that the solution for r_1 and r_3 is made with the new values of the thicknesses, and that r_2 is taken from Table 2.

On these assumptions it has been found that changes in the residual spherical aberration produced by changing either thickness from the standard value to zero will not exceed 0.02 and will usually be considerably less than this amount. Thus for such variations in thicknesses as may be required in the work of practical designing the effects on spherical aberration may be ignored.

Moreover, it follows from the above that the choice of glasses would be unaffected by ordinary thickness changes, since the height of the spherical aberration maximum is the factor that determines the choice of glasses, and since it is practically unaffected by ordinary thickness changes.

Coma residuals are somewhat more sensitive to thickness changes, although the effects are still negligible in many examples of practical designing. Reducing $t_{C'}$ from the standard value to zero sends the coma residual 0.02 to 0.03 in the direction of positive values. While there is considerable variation in this coefficient over the domain represented by Table 2, the average value just stated ought to serve very well since the effects are almost negligible anyway in the majority of cases.

However, if one desires to make correction for the small coma residual likely to be introduced by thickness changes, it can be readily done by applying a correction to the tabular value of r_2. The coefficient just given together with the known slope (slope = 40) of the coma line enables one to calculate the necessary correction to the value of $1/r_2 = \rho_2$ that will apply in any specific example. Since the effects are so small the assumption of proportionality is satisfactory.

Changing t_{fl} from the standard value to zero produces effects on the coma residual similar to those produced by a similar change in t_{cr}, but of about one third the magnitude as an average coefficient.

Effects of Approximate Glasses.—In actual practice the designer will usually find it impossible to select a pair of glasses that precisely meets the dictates of Fig. 3 and Table I. He is then confronted with the question of how much a small departure from the ideal values will affect the residual aberrations of the adjusted combination. One might proceed in various ways here, but two methods stand out as being most suitable. It may be assumed that spherical aberration and coma are to be fully corrected with chromatic aberration coming as it must under the circumstances, or it may be assumed that chromatic aberration is to be corrected while the balance of coma and spherical aberration is adjusted to make the best compromise. Both are useful and both will now be discussed.

Thus far, only those combinations that give precisely the same equivalent focal length for the *C* and for the *F* lines have been considered. Such precise adjustment of chromatic aberration is seldom necessary and frequently undesirable. For instance, if there is to be some prism glass in the image space of the objective the designer will usually want to leave enough chromatic aberration in the objective itself to be counter-balanced by that of the prism glass. Or, if some compensation between objective and ocular is intended the adjustment of the chromatic aberration of the objective would be made in accordance.

Moreover, in those cases in which precise achromatization for *C* and *F* would be desirable, it will not always be possible to select a pair of glasses in strict accordance with the dictates of Fig. 3. The pairs of glasses either available or procurable will usually depart, at least a little, from these conditions. Therefore, it is desirable to be able to ascertain quickly how much chromatic aberration a given pair of available or procurable glasses will give when the lens is adjusted to eliminate spherical aberration and coma in precisely the same manner as it would be done if the glasses had the Δn ratio called for in Fig. 3, in accordance with the indices of refraction of the glasses.

To derive an approximate expression for the residual aberration when the actual Δn ratio departs from that called for in Fig. 3, and when the radii are determined precisely as they would be if the ideal Δn ratio did hold, one may proceed as follows according to the principles of thin lens geometry, which are sufficiently precise for the purpose.

The powers of a thin two lens contacted combination for the colors designated by the C and by the F lines respectively, are

$$\left.\begin{aligned}\varphi_C &= k_1(n_C-1)+k_2(n_{2C}-1)\\ \text{and}\quad \varphi_F &= k_1(n_{1_F}-1)+k_2(n_{2_F}-1)\end{aligned}\right\} \tag{14}$$

Let $\Delta\varphi$ be the chromatic residual of power, and if it is to be positive when the longitudinal chromatic aberration is positive (over-correction), then it must be defined as

$$\Delta\varphi=\varphi_C-\varphi_F \tag{15}$$

But

$$\varphi_C-\varphi_F=k_1(n_{1_C}-n_{1_F})+k_2(n_{2_C}-n_{2_F}) \tag{16}$$

Therefore

$$\Delta\varphi=-k_1\Delta n_1-k_2\Delta n_2 \tag{17}$$

Now, in the case for which $\Delta\varphi=0$,

$$k_1=\frac{\varphi\Delta n_2'}{\Delta n_2'(n_1-1)-\Delta n_1'(n_2-1)}\quad:\quad k_2=\frac{-\varphi\Delta n_1'}{\Delta n_2'(n_1-1)-\Delta n_1'(n_2-1)}. \tag{18}$$

in which the primes ($'$) indicate the ideal value of the Δn's according to Chart A, that give $\Delta\varphi=0$, and the φ's and the n's refer either to the C or to the F line.

Substitution of these values in Equation (17) gives:

$$\Delta\varphi=\varphi\left[\frac{\Delta n_2'\Delta n_1-\Delta n_1'\Delta n_2}{\Delta n_1'(n_2-1)-\Delta n_2'(n_1-1)}\right]. \tag{19}$$

Or

$$\Delta\varphi=\frac{\varphi}{(n_2-1)-\dfrac{\Delta n_2'}{\Delta n_1'}(n_1-1)}\cdot\left(\frac{\Delta n_2'}{\Delta n_1'}\Delta n_1-\Delta n_2\right) \tag{20}$$

As a typical example of the application of this formula take the pair of glasses:

$n_D=1.5745$ $\quad\Delta n=0.01000$

Δn ratio $=1.724$

$n_D=1.62177$ $\quad\Delta n=0.01724$

The ideal Δn ratio as read from Fig. 3, or Table 1, for the two indices of refraction specified is 1.766. Substitution of these values in the formula for the determination of $\Delta\varphi$ and reduction to difference of equivalent focal length gives a residual chromatic error of -0.107 for an equivalent focal length of 100. That is, the spherically and comatically corrected

objective of these glasses would be under-corrected by this amount for chromatic aberration. It is thus seen that a pair of glasses may depart appreciably in Δn ratio from the dictates of Fig. 3, or Table 1 before seriously interfering with the performance of the resulting objective with respect to chromatic aberration. This is indeed fortunate for it gives considerably more freedom in the pairing of glasses than is indicated in the precise Fig. 3. In fact, this might be offered as a criticism of the method. However, when the workings of the system with respect to chromatic aberration are understood there should be no difficulty. It simply means that the chromatic aberration is much less sensitive to changes in the Δn ratio than are the spherical aberration and the coma. It seems best therefore to control these aberrations most affected by the variations of the Δn ratio and let the less sensitive chromatic aberration take care of itself more or less.

An alternative procedure would be to plan on holding the chromatic correction by using the actual Δn ratio in the achromatic solution and then to determine what errors in spherical aberration and in coma would arise from small departures in $(n_{Fl}-n_{Cr})_D$ from the values obtained by the normal use of Fig. 3 and Table 1. To do this, the effects of $(n_{Fl}-n_{Cr})_D$ change on spherical aberration and on coma must be known. For combinations having a Δn ratio around 2.4, an increase of 0.001 in this quantity will move the spherical aberration peak about 0.008 toward over-correction, and will send the coma in the direction of negative values by about 0.004. For a Δn ratio of 1.4 and an $(n_D-1)_{Cr}$ of 0.46 an increase of 0.001 in the values of $(n_D-1)_{Fl}$ will move the spherical aberration about 0.10 in the positive direction and the coma about 0.025 in the negative direction. At $(n_D-1)_{Cr}=0.64$ on the 1.4 line the same change would move the spherical aberration in the positive direction by about 0.03 and the coma negative by about 0.012. As a sufficiently good approximation it may be assumed that the change is gradual along the Δn ratio lines. These coefficients will enable one to estimate the effect of slight departures in the glass data from the values obtained by the use of Fig. 3 and Table 1.

Illustrative Example. In order to efficiently perform a complicated calculation, such as the achromatic solution for r_1 and r_3, it is necessary to have a systematic plan for the work. The following example shows a good arrangement for the routine solution of the problem. It has been found to be quite satisfactory, although no doubt it could be improved. Much of the speed and accuracy with which a skilled computer makes an involved calculation comes as a result of having given

due regard to good form in the arrangement of the work. This point cannot be overemphasized for it is one likely to be disregarded by beginners, partly because they have no background of experience to guide them in the arrangement of the work, and partly because they do not realize that much mental fatigue may come from a procedure that leaves too many things to think about during the course of the calculations.

Let it be assumed that the light barium crown glass.

$$n_D = 1.5683 : \Delta n = 0.00995,$$

was available, and that it was desired to find what flint glasses, if any, were available that would combine with this barium crown to form a coma-free cemented telescope objective, and to design the lens.

By the use of the method already described the locus of suitable flint glasses was drawn on the glass chart. Two available ordinary flints were found close to the curve. The one selected was $n_D = 1.5810$; $\Delta n = 0.01421$. The Δn ratio was thus 1.43, and by interpolation in Table 2 the proper value of r_2 was found to be -21.06. The achromatic solution for r_1 and r_3 was then carried out by machine as recorded in the double column at the right hand side of the page of calculation reproduced below with a suitable key. The next step was to find the equivalent focal length and the position of the principal points for the D line. Many designers prefer the zero ray method for this determination. The method here shown has merits of its own, however, inasmuch as it does not require the use of tables and can be done entirely on a machine. Moreover, it gives the position of both principal points. It will be noticed that the equivalent focal length for D is slightly less than 100, as is to be expected as a result of secondary spectrum. In fact, this calculation may be taken as a check on the achromatic solution for r_1 and r_3. The remainder of the page is the well known tracing of an $h = 7$ ray for the D line to check the spherical aberration and the coma. If it were necessary, the coma could be brought very close to zero by reducing the numerical magnitude of ρ_2 by 1/40th of the amount of the coma residual and using this new ρ_2 in an achromatic solution for r_1 and r_3. This follows from the fact that the slope of the coma curve in Fig. 1 is always approximately 40. However, since the result is entirely satisfactory as it stands for practical purposes, such a readjustment would probably not be made. Therefore, the entire designing of this objective is recorded on a single standard letter size page.

Key to the Calculation

Single Lenses D

r_1	$-r_2$	$-t_1$	n_1
$a=r_1/(n_1-1)$	$b=-r_2/(n_1-1)$	$c=-t_1/n_1$	$\Sigma=a+b+c$
$f_1=ab/\Sigma$		$p_1=f_1c/b$	$p_1'=f_1c/a$
r_2	$-r_3$	$-t_2$	n_2
$a=r_2/(n_2-1)$	$b=-r_3/(n_2-1)$	$c=-t_2/n_2$	$\Sigma=a+b+c$
$f_2=ab/\Sigma$		$p_2=f_2c/b$	$p_2'=f_2c/a$

Combination D

Air-space

p_1'

p_2

f_1	f_2	$c=p_2+p_1'+\text{Air}$	$\Sigma=f_1+f_2+c$
$F=f_1f_2/\Sigma$		$g=fc/f_2$	$h=fc/f_1$
P'		p_1	p_2'
$Bf=f+p'$		$P=g+p_1$	$P'=h+p_2'$

h=7 Ray D

	$-u$	$-u$
r_1	r_2	r_3
	r_2-u	r_3-u
$\log h$	$\log(r_2-u)$	$\log(r_3-u)$
$\operatorname{colog}(-r_1)$	$\operatorname{colog}(-r_2)\rho$	$\operatorname{colog}(-r_3)$
	$\log\sin U'_{k-1}$	$\log\sin U'_{k-1}$
$\log\sin(-i)$	$\log\sin(-i)$	$\log\sin(-i)$
$\log(-n/n')_1$	$\log(-n/n')_2$	$\log(-n/n')_3$
$\log\sin i'$	$\log\sin i'$	$\log\sin i'$
$-i$	$-i$	$-i$
i'	i'	i'
$i'-i$	$i'-i$	$i'-1$
U	U	U
$U'=i'-i+U$	$U'=i'-i+U$	$U'=i'-i+U$
$\log\sin U'$	$\log\sin U'$	$\log\sin U'$
$\operatorname{logcsc} U'$	$\operatorname{logcsc} U'$	$\operatorname{logcsc} U'$
$\log(-r_1)$	$\log(-r_2)$	$\log(-r_3)$
$\log\sin i'$	$\log\sin i'$	$\log\sin i'$
$\log(-N)$	$\log(-N)$	$\log(-N)$
$-N$	$-N$	$-N$
r_1	r_2	r_3
$u'=r_1-N$	$u'=r_2-N$	$u'=r_3-N$
$-t_1$	$-t_2$	$-$Back focus.
$u(\text{next})=u'-t_1$	$u(\text{next})=u'-t_2$	Spher. Aber.
$\log h$		Sin Con Error
$\operatorname{logcsc} U'_2$		
log (Sin. Con.)	Sin. Con.	Coma
	$-F$	$=$S.C.E.$-$S.A.
	Sin. Con. Error	

General Information

$-r_2$

$-\rho_2$

C line	*F line*
n_1	n_1
n_2	n_2
n_2-n_1	n_2-n_1
$-\rho_2(n_2-n_1)$	$-\rho_2(n_2-n_1)$
t_1	t_1
t_1/n_1	t_1/n_1
t_1-t_1/n_1	t_1-t_1/n_1
A Eq. (9)	*A* Eq. (9)
t_2	t_2
t_2/n_2	t_2/n_2
t_2-t_2/n_2	t_2-t_2/n_2
$-B$ Eq. (10)	$-B$ Eq. (10)
C Eq. (11)	*C* Eq. (11)
D Eq. (12)	*D* Eq. (12)

Quadratic equation and its solution for r_1.

Solutions for r_2. Eq. (13)

Calculation of A Coma-Free Objective

Single Lenses D

64.3973	21.06	−3.5	1.5683
113.3157	37.0579	−2.2317	148.1419
28.3461		−1.7071	−0.5583
−21.06	318.2731	−1.75	1.5810
−36.2478	547.8022	−1.1069	510.4475
−38.9004		0.0786	−1.1879

Combination D

		0.0000	
		−0.5583	
		0.0786	
28.3461	−38.9004	−0.4797	−11.0340
F_D=99.9343		1.2323	−1.6912
−2.8791		−1.7071	−1.1879
BF_D=97.0552		−0.4748	−2.8791

k=7 Ray D

	−173.7862	−187.2219
64.3973	−21.06	−318.2731
	−194.8462	−505.4950
0.845098	2.289692—	2.703717—
8.191132—	8.676542	7.497200—
	8.597018—	8.560922—
9.036230—	9.563252	8.761839
9.804571—	9.996497—	0.198932—
8.840801	9.559749—	8.960771—
−6 14 25.4	21 27 26.1	3 18 46.3
3 58 27.9	−21 16 35.2	−5 14 31.3
−2 15 57.5	0 10 50.9	−1 55 45.0
0	−2 15 57.5	−2 05 06.6
−2 15 57.5	−2 05 06.6	−4 00 51.6
8.597018—	8.560922—	8.845135—
1.402982—	1.439078—	1.154865—
1.808868—	1.323458	2.502800
8.840801	9.559749—	8.960771—
2.052651	2.322285	2.618436

Δn Ratio=1.43

Crown Glass:

n_D=1.5683

Δn=0.00995

Flint Glass:

n_D=1.5810

Δn=0.01421

$-r_2 = -21.06$

$-\rho_2 = 0.047483$

C	*F*
1.56539	1.57534
1.57700	1.59120
0.01161	0.01586
0.00055128	0.00075308
3.5	3.5
2.2358645	2.2217426
1.2641355	1.2782574
0.5660869	0.5763026
1.75	1.75
1.1097020	1.0997989
0.6402980	0.6502011
0.5773530	0.5916897
1.0918705	1.1304184
0.01055128	0.01075308

$$106.672397\rho_1^2$$

$$-24.0414324\rho_1$$

$$+0.3476070=0$$

$$\rho_1 = \frac{3.312945}{213.34479}$$

$r_1 = 64.397322$

Calculation of A Coma-Free Objective—*cont.*

112.8889	210.0319	415.3709
64.3973	−21.06	−318.2731
177.2862	188.9719	97.0978
−3.5	−1.75	−97.0552
173.7862	187.2219	S.A. = +0.0426
0.845098		S.C.E. = +0.057
1.154865		
1.999963	99.9915	Coma = +0.014
	−F = −99.9343	
	S.C.E. = +0.0572	

$$r_{3C} = \frac{-36.088117}{.11338728}$$

$$= -318.27306$$

$$r_{3F} = \frac{-36.972814}{.11616696}$$

$$= -318.27306$$

$$r_3(\text{mean}) = -318.2731$$

CONCLUSION

In conclusion, it may be pointed out that what has been done for the two lens cemented telescope objective might possibly be done in somewhat similar fashion for other optical systems. It is likely that many a designer has performed far more actua' calculation in the pursuit of his routine designing of a limited number of objectives than would be necessary to investigate a whole domain. And yet, he may have discovered few of the fundamental and helpful empirical laws that underly the subject and are there for use at the cost of finding them.

Not only do investigations of this sort help in the designing of optical systems, but they may be of value in other directions as well. For instance, a broad and exhaustive survey of a problem in designing may point to the desirability of obtaining glasses on certain portions of the glass chart now vacant, or to stress the desirability of getting a greater variety of glasses in some particular portion of the glass chart. Here, as in other fields of research, studies in one field suggest new studies in allied fields.

OPTICAL LABORATORY,
FRANKFORD ARSENAL,
PHILADELPHIA, PA.
DECEMBER, 1924.

THE EFFECT OF TRANSVERSE JOINTS ON THE MAGNETIC INDUCTION IN NICKEL

By L. W. McKeehan

In a recent paper S. R. Williams[1] describes some experiments on the behavior of piles of nickel plates in a magnetic field and concludes that the results agree with "the assumption that above a certain critical field strength the surfaces of narrow slits cut in nickel normal to the magnetizing force are more susceptible than are the interior layers." It is unnecessary, however, to assume that nickel possesses a property so peculiar and difficult of explanation, the experimental results reported being explicable without recourse to this artifice, if due account is taken of the probable state of the material under test. A review of the data shows that the result which the author ascribes to an excess of susceptibility across the air-gaps in high fields can be as readily explained by an excess of susceptibility parallel to the sheets in low fields. The latter explanation is consonant with the known dependence of susceptibility upon the state of mechanical stress.

Nickel sheets prepared in the usual way by rolling are found to be in a state of tension along the surface and correspondingly the internal layers are in compression parallel to the surface. This type of stress-distribution will persist even after moderate annealing. The polishing of the sheets, to which the author refers, also tends to set up stress-distributions of the same type. In both cases the intensity of stress in the surface layer is much higher than that in the interior so that we may picture the sheet as composed of two thin sheets stretched over a thicker central portion. It is well known that the effect of compression in nickel is to increase its magnetic susceptibility, the increase being greater proportionately in weak fields. In sheets of the sort described, therefore, a large part of the material will initially be more susceptible in directions parallel to the surface than in a direction normal thereto.

This is all that is necessary to explain the observed behavior of approximately cubical piles of square pieces. In piles like those described by the author, in which the thickness is slightly greater than the transverse dimension and in which the field is so applied as to make equal angles to the plane of the sheets and to their normal, the excess of initial permeability parallel to the surface of the sheets will result

[1] J.O.S.A., *10*, pp. 109-118; January 1925.

in a greater magnetization in this direction during the first stages, while at the conclusion of the process the greater magnetic moment, possible when the normal is parallel to the applied field, will result in saturation along this line. The effect of external pressure in a direction perpendicular to the plane of the sheets is, of course, to increase the susceptibility in that direction in low fields and therefore to diminish the tendency for the planes to turn initially into parallelism with the applied field. This is well shown in Figs. 5 and 6 of the article referred to.

The earlier experiments of E. H. Williams[2] are not directly comparable with those of S. R. Williams and it may be noted that the difference between iron and nickel in high fields, which E. H. Williams observed and attributed to the different effects of transverse joints in the two metals, was very small and may have been due to secondary causes such as differences in smoothness.

In iron the effect of compression is to reduce the initial susceptibility so that in piles of iron sheets the rotation is continuously in one direction, as reported by the author.

Experiments have shown that permalloy is a true solid solution of iron (and small amounts of other metals) in nickel[3] and that both the iron and the nickel become magnetically saturated in low fields, so that S. R. Williams' suggestion that the abnormal properties of these alloys are due to a property of the nickel alone appears unlikely.

Bell Telephone Laboratories, Incorporated,
New York, N. Y.
March 12, 1925.

Increase of thermionic Current due to Heating of the emitting Metal by absorbed Light.—Allowing an intense concentrated beam of unresolved white light interrupted 1000 times per second to fall upon a hot tungsten filament in a tube, Henriot and Moens heard a tone of frequency 1000 in a telephone receiver connected in the exterior circuit between the filament and the plate. This was accompanied by a rise in the resistance of the filament indicating a temperature-rise of the same order as would produce the observed extra current by virtue of mere increase in thermionic emission, so that no true photoeffect need be postulated. Their criticism of the work of Arnold and Ives on the true photoeffect observed with oxide-coated filaments is devoid of foundation.—[E. Henriot and R. Moens; C. R. *180*, pp. 651-653; 1925.]

Karl K. Darrow

[2] Physical Review, *33*, pp. 60-64; 1911.
[3] McKeehan, L. W., Physical Review (2), *21*, pp. 402-407, April 1923.

THE GEOPHYSICAL LABORATORY FURNACE THERMOSTAT

By Howard S. Roberts

This thermostat was developed at the Geophysical Laboratory to maintain electrically heated furnaces at a constant temperature. Its basic feature, which is due to W. P. White,[1] is the use of the heating coil of the furnace itself as a resistance thermometer. By keeping the resistance of this coil constant, the thermostat maintains a constant mean temperature of the coil. Thus a constancy of the furnace cavity as a whole to 0.1° has usually been realized as far up as 1200°C without attention. At higher temperatures a slow downward drift of temperature may easily be corrected manually.

Several thermoregulators of this type have been in use in the Laboratory for some years; they have been modified and improved from time to time,[2] so that it now seems worth while to present a description, in sufficient detail that anyone may be able to construct a workable thermostat; and to discuss some of its principles, uses, and limitations.

In addition to its greater reliability and sensitivity, this method of control possesses two outstanding advantages over the method in which a thermometer of some sort is placed inside the furnace. (1) No part of the thermostat is placed inside the furnace, unless we consider the heating coil of the furnace as a part of the thermostat. (2) Since the heater and thermometer are identical, hunting—a slow oscillation of temperature due to the thermal lag between heater and thermometer—does not occur. In the original thermostat of White and Adams,[3] having mechanically operated galvanometer contacts, it was theoretically possible for hunting to be brought about by the action of these contacts;[4] but this was not realized experimentally.

The method is subject to the following limitations: In order that the temperature of an object within the furnace shall remain constant it is necessary (1) that the resistance of the heater change appreciably with temperature, and (2) that there be a close correspondence between the temperature of the object and the resistance of the heater. The first condition is met satisfactorily by most of the metals and alloys

[1] White and Adams, Phys. Rev., *14*, pp. 44-48; 1919.

[2] Howard S. Roberts, J. Wash. Acad. Sci., *11*, p. 401; 1921; J. Opt. Soc. Amer., *6*, p. 965; 1922; Leason H. Adams, J. Opt. Soc. Amer., *9*, pp. 599-603; 1924.

[3] Op. cit.

[4] Roberts, J. Wash. Acad. Sci., *11*, p. 405; 1921.

that are likely to be used as heating coils: the thermostat has been used successfully with heaters of platinum, platinrhodium, molybdenum, nickel, and chromel. The second condition implies, first, that the relation between the temperature of the heater and its resistance remain sensibly fixed, that is, that the heater shall not deteriorate rapidly; and, second, that the temperature of the object be very little influenced by factors other than the temperature of the heater. Deterioration of the heater at temperatures up to about 1200° can be made negligible by making the heater of platinum. The second consideration is subject to the same conditions that must be met to secure uniformity in the temperature distribution within the furnace, the most important of which is that the thermal resistance between the heater and the object

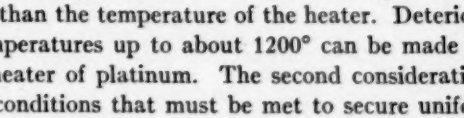

FIG. 1.—*Abridged diagram showing the basic principle of the thermostat.*

to be heated shall be low in comparison with that between the object and the surroundings of the furnace. In the case of a cylindrical furnace, for example, this may be attained by making the length of the heated tube ten or more times its diameter.[5]

GENERAL PRINCIPLES OF THE THERMOSTAT

The basic principle of the thermostat may be understood from Fig. 1. A furnace is shown whose heater has an appreciable temperature coefficient of resistance; it is associated with other resistances, A, B, C, whose temperature coefficients are negligibly small, to form a Wheatstone bridge. These resistances, which are adjustable, are given such values that the bridge is balanced when the furnace resistance is that corresponding to the temperature at which it is to be held. With the furnace at this temperature no current flows in the galvanometer circuit xDy,

[5] See J. B. Ferguson, "Temperature uniformity in an electric furnace." Phys. Rev., *12*, pp. 81-94; 1918.

but a change of temperature in either direction unbalances the bridge and produces a current in this circuit, its direction depending on whether the temperature has become higher or lower than that for which the bridge was balanced

The current from the line passes through a fixed resistance R and through a rheostat Rh, the latter being adjusted so that, with R in the circuit, too little current flows, and with R shunted, by the switch E, too much current to maintain the furnace at the desired temperature. A suitable device at D, which includes a galvanometer actuated by the current in the "galvanometer circuit" of the bridge, opens and closes the switch E as the resistance of the furnace becomes too high and too low respectively. When E opens, the current through the furnace is reduced, and the temperature and the resistance of the heating coil decrease. Similarly, when E closes, the temperature and the resistance of the heating coil increase. Thus the resistance of the heating coil is caused to oscillate through a short interval on either side of some definite value, its average resistance remaining constant. The corresponding variation in the temperature of the wire of which the heating coil is made often amounts to several degrees; but the period of oscillation is seldom more than a few seconds and, because of lag, its effect on the constancy of temperature inside the furnace is usually too small to measure by ordinary methods.

DETAILS OF ITS CONSTRUCTION AND OPERATION

Fig. 2 is the complete diagram, and Fig. 3 a photograph, of one of the thermostats in actual use. This particular thermostat is used with either direct or alternating current at 110 volts and is arranged to control any suitable furnace that consumes not more than 15 amperes heating current and has a resistance, when in use, of 1.5 to 40 ohms.

In Fig. 2, as was the case in Fig. 1, the heating coil F of the furnace forms one arm of the Wheatstone bridge $wxzy$. By means of the sliding contacts x and y the bridge may be balanced, that is, the contacts x and y brought to the same potential, when the resistance of the furnace has any value between chosen limits. The current supply for the bridge passes through a rheostat Rh and a fixed resistance R, corresponding to Rh and R in Fig. 1. Contacts x and y are connected to the galvanometer G through the synchronous rectifier S, which may be short-circuited by the switch Sw when direct current is being used.

The insulated boom b of the galvanometer is arranged to make either of the two contacts a and c, depending on the direction of the current

through the galvanometer coil. The boom is connected to the grid of a triode tube T whose plate current passes through a high resistance relay R_1. The potentials of the contacts a and c are so chosen that when the galvanometer closes contact c very little current flows in the plate-filament circuit of the triode and relay R_1 is open; while when the galvanometer closes contact a a moderate plate current flows and the relay closes. The closing of R_1 causes R_2 also to close and to short-circuit the resistance R in series with the bridge and furnace.

It will be well at this point to describe the operation of the thermostat, leaving the more detailed description of its parts until this is

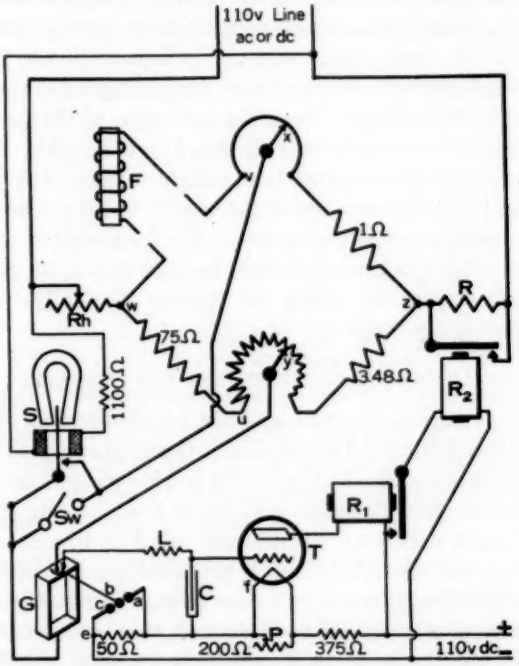

FIG. 2.—*Complete diagram of the connections between all of the essential parts of the 110 volt, 15 ampere, ac or dc thermostat.*

understood. We shall assume that the furnace is hot and the bridge balanced; that the rheostat is adjusted so that with R in the circuit too little current flows, and with R short-circuited, too much current flows to maintain the furnace at its present temperature; and that the relay and galvanometer contacts are in the positions shown in Fig. 2.

Since the relay R_2 is open the extra resistance R is in circuit and the furnace is cooling; the bridge becomes unbalanced and b swings toward the right. When contact b reaches contact a the grid of the triode T is brought to the potential f which causes the plate current to increase and close the relay R_1. The closing of R_1 sends a current through R_2, causing it to close. The extra resistance R is now short-circuited by R_2, causing the furnace temperature to rise. This makes b move toward the

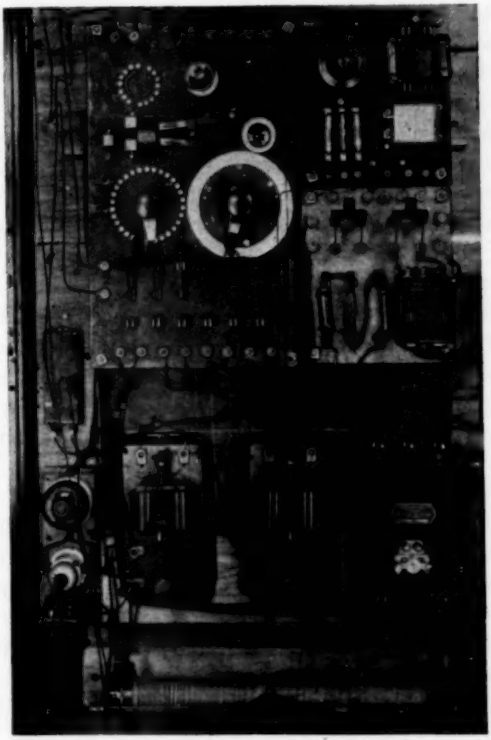

FIG. 3.—*Arrangement of the various parts. The bridge resistances and series lamp bank are not included in the picture.*

left; when it reaches c the grid is brought to the negative potential e, which is sufficient practically to stop the flow of current in the plate circuit. As a result R_1 and in turn R_2 open, restoring the extra resistance R. This causes the furnace to cool and b again swings toward the right, completing one cycle.

Bridge and Resistors. The bridge is composed of a group of coils having, with the exception of the furnace coil, a negligibly small temperature coefficient of resistance. From the standpoint of minimizing parasitic thermal emf's in the galvanometer circuit it is desirable either that the coils be wound with an alloy, such as manganin or therlo, which has in addition to its low temperature coefficient a low thermal emf against copper and brass; or that each coil be arranged so that its ends are at approximately the same temperature, when constantan, advance, ideal, etc., may be used. Since the bridge is to be used with alternating current it is desirable that the power factors of the coils be high and of the same order of magnitude as that of the furnace.[6] We approximated this condition by winding the larger coils on alundum tubes similar to those used in most of our furnaces, the smaller coils being short wires stiff enough to support themselves.

The galvanometer circuit is connected to the bridge through two sweep contacts x and y. Contact x is movable along a length of No. 16 B & S gage "Advance" wire bent into an arc, and consists of a short piece of the same wire soldered to the under side of a brass arm pivoted at the center of the arc formed by the slide wire. The slide wire is 32 cm long and has a resistance of about 0.12 ohm.

Contact y, a brass arm, makes contact with any one of a circle of 29 brass buttons connected to the junction points of a series of coils. The resistances of these coils are such that a geometric series is formed by the sums of the resistances between successive contact buttons and the corner z of the bridge (Fig. 2). The ratio of any term of the series to the preceding term is a constant a little less than the ratio of the one-ohm coil plus the slide wire to the one-ohm coil, and in this case is equal to $10^{1/24}$ (that is, 1.101). By using this ratio the series repeats itself, with a shifted decimal point, after 24 terms, making it possible to extend it indefinitely by inspection. The series, and the resistances of the individual coils in this tract are given in Table 1.

With the values of resistance given in Fig. 2 and Table 1, the range of allowable furnace resistance will be from 1.5 to 40 ohms. If, in place of a geometric series, a set of equal coils had been used, it would have been necessary, leaving the 3.48 ohm coil as it is, to make each equal to the present 0.351 ohm coil. If this had been done, the range, with the same number of contact buttons, would have been only from 5.6 to 27 ohms.

[6] The effect of power factor on the accuracy of temperature control by this thermostat is discussed by Roberts, J.O.S. A. & R.S.I., *6*, p. 969; 1922.

The range of furnace resistances may, of course, be shifted either way by changing the bridge ratios. For example, if a shunt of 1.12 ohms is connected around the slide wire and one-ohm coil, that is, from v to z in Fig. 2, the upper and lower limits will be halved. Other methods will suggest themselves, but it must be borne in mind that the ratio of the

TABLE 1. *Values of resistance coils under contact y.*

Button	Total	Coil	Button	Total	Coil
0	3.481		15	14.68	
		.351			1.478
1	3.831		16	16.16	
		.386			1.627
2	4.217		17	17.78	
		.426			1.791
3	4.642		18	19.57	
		.468			1.970
4	5.109		19	21.54	
		.514			2.170
5	5.623		20	23.71	
		.566			2.388
6	6.190		21	26.10	
		.623			2.628
7	6.813		22	28.73	
		.686			2.893
8	7.499		23	31.62	
		.755			3.184
9	8.254		24	34.81	
		.831			3.506
10	9.085		25	38.31	
		.915			3.857
11	10.000		26	42.17	
		1.007			4.256
12	11.007		27	46.42	
		1.108			4.674
13	12.11		28	51.09	
		1.220			
14	13.34				
		1.343			

The second column gives the series formed by the sums of all the coils between successive contact points and the corner z of the bridge (Fig. 2). The third column gives the resistance of the individual coils.

furnace resistance to that of the coil which in the present apparatus is the one-ohm coil can not be made very great without considerable loss in sensitivity. From this standpoint a useful compromise between sensitivity and power lost in the "one-ohm" coil lies in a choice of ratios between 5:1 and 15:1.

Where the furnace is to be used for making heating or cooling curves, a convenient method of changing the balance point of the bridge is the use of a variable shunt in parallel with the 75 ohm coil of the bridge. For the shunt, we have used a two-dial resistance box having a total resistance of 990 ohms in steps of 10 ohms; this is connected to the points *u* and *w*, Fig. 2, through a 10 ohm rheostat and an additional 1000 ohm resistance. For convenience the resistance box and rheostat are placed at the side of the potentiometer used in measuring the temperature of the furnace; thus the operator is enabled to control the temperature of the furnace without leaving the potentiometer. A sample heating curve obtained with the aid of this device is shown elsewhere.[7]

In order to decrease sparking at the contacts of relay R_2 the extra resistance R is preferably a bank of 32 volt Mazda C (tungsten filament, gas-filled) lamps. The resistance of the cold filament of such a lamp is only about one-fifteenth of its resistance when heated to full candle power. Thus, at the instant the relay contacts open, the filament is cool and the RI drop through it is small (usually of the order of 5 volts). As this is the voltage actually broken by the contacts, there is very little sparking, although the resistance increases and the voltage rises to its normal value immediately after the contacts have separated. Lamps of higher wattage than 100 are not desirable for this purpose because the filament cools too slowly, the 100 watt filament being still a dull red one second after the relay has closed. It is obvious that, by connecting additional lamps, the voltage absorbed by the lamp bank may be reduced to any point with a corresponding decrease in the voltage broken by the relay contacts. This is desirable, if the variations in line voltage are small enough to permit it, because it is more economical of power and because a more constant furnace temperature may be obtained.

Rectifier and Galvanometer. The synchronous rectifier S is shown, drawn to scale in Fig. 4. In it a reed R made of clock spring, whose free period is about equal to the frequency of the alternating current, is arranged with its free end between the poles N and S of a permanent magnet. A coil C of about 1500 turns of No. 36 B & S gage copper wire is wound on a hollow bobbin of vulcanized fiber within which the reed is free to swing. This coil is connected through a resistance, such as a 10 watt lamp, to the alternating current line and induces an alternating magnetic flux in the reed. The end of the reed is attracted alternately by the north and by the south pole of the magnet and is thereby

[7] Roberts, J. Wash. Acad. Sci., *11*, p. 408; 1921.

caused to vibrate with the frequency of the alternating current. Two stops, one of which is a contact, are provided to limit the motion of the reed and to make it work more smoothly.

As orginally conceived a specially constructed transformer[8] having two secondary windings was inserted in the galvanometer circuit between the bridge and the rectifier, and the reed arranged with two contacts to rectify both halves of the alternating current wave. This arrangement gave somewhat more sensitivity, but it was found that the transformer could be dispensed with, as is the case in Fig. 2. It might, however, be reinstated to step up the voltage of the galvanometer circuit where the furnace and bridge are to be used on a low voltage.

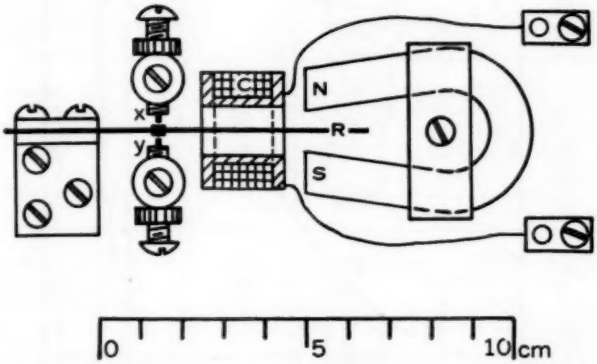

FIG. 4.—*Detail of vibrating synchronous rectifier shown at S in Fig. 2.*

The galvanometer *G* in Fig. 2 is a Weston "Model 30, five binding post galvanometer" having a resistance of about 50 ohms and a period of about 2/3 second. Its sensitivity, which has been found ample, is a movement by the contact button of about 1/2 mm for 1 millivolt. During the early history of the thermostat considerable annoyance was caused by sticking of the galvanometer contacts. Various devices[9] were tried to discourage sticking and to separate the contacts automatically after they had stuck. None of these was wholly satisfactory until we introduced a triode tube as an intermediate relay. The advantage of such a relay lies in the fact that its operating current, which must pass through the galvanometer contact, may be made less than 1 micro-

[8] Roberts, J.O.S.A. & R.S.I., *6*, p. 965; 1922.

[9] Ro berts, J.O.S.A. & R.S.I, *6*, p. 965; 1922.

ampere. This solved the difficulty of sticking and simplified the rest of the apparatus.

Triode and Relays. The principle of the triode may be understood from the characteristic curves of a particular Radiotron UV-201A in Fig. 5. In curve P the plate current I_P in milliamperes, and in curve G the grid current I_G in microamperes are plotted against the potential of the grid with respect to the negative filament terminal. The plate potential was 100 volts positive with respect to the negative filament terminal and the voltage across the filament terminals was 3.5, this voltage having been found sufficient for our purpose.

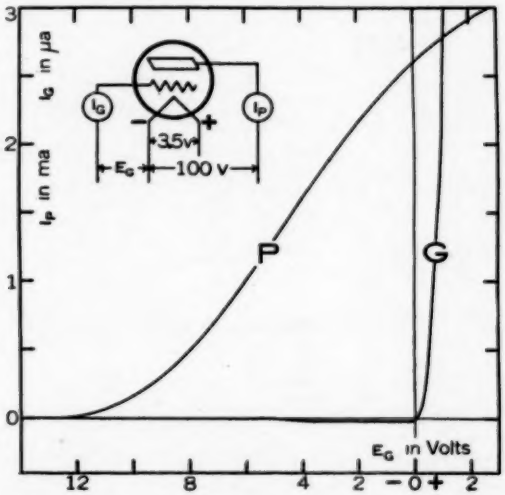

FIG. 5.—*Plate current and grid current characteristics of the triode.*

The conditions to be met by the triode are (1) a very small operating (grid) current, and (2) practically an "on and off" output (plate) current since this is to operate a relay. As is evident from the figure, these conditions are satisfied if we set the grid potential at about 0 volts to close the relay and at about −12 volts to open it. When operating between these two values of grid potential, the grid current, which must pass through the galvanometer contacts, is never more than 0.03 microampere, while about 2.5 milliamperes is available in the plate circuit to operate the relay. The life of the triode should be considerably lengthened by keeping the filament temperature as low as is consistent with reliable operation of the relay. We have found a filament current

of about 0.2 ampere at 3.5 volts ample for the Radiotron UV-201A when new, and, presumably as a result of this low filament current, one of these tubes has already given more than 2000 hours of service without our having to increase the filament current. The grid potential should not be allowed to become positive: as may be seen in Fig. 5, little is to be gained in plate current by making the grid positive while there is a very rapid increase in the grid current, which must pass through the contacts of the galvanometer.

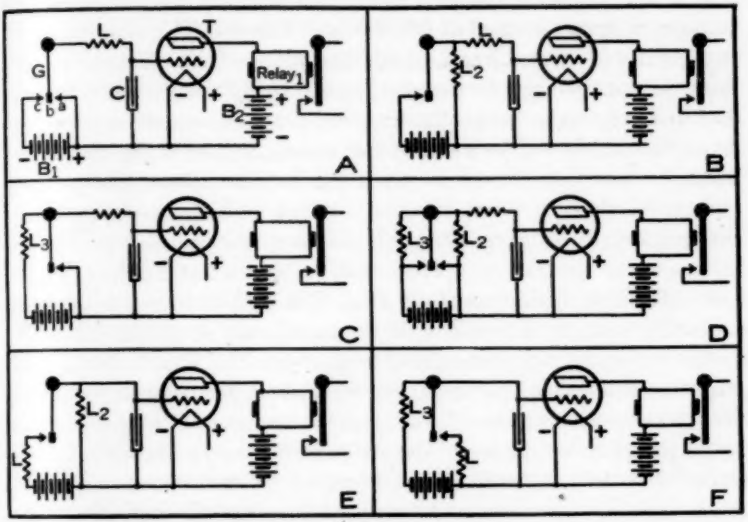

FIG. 6.—*Several methods of using the triode as an intermediate relay.*

Fig. 6A is substantially the arrangement of galvanometer, triode and relay already shown in Fig. 2. One contact, *a*, of the galvanometer is connected directly to the negative filament terminal while the other, *c*, is maintained at a potential 12 volts lower by means of the biasing battery B_1. With the boom, *b*, of the galvanometer touching the right hand contact *a*, the grid potential is zero and sufficient current is flowing from the battery B_2 through the relay in the plate circuit to close the relay. With the boom touching the left hand contact the grid potential is about -12 volts and the plate current is too small to keep the relay closed. Because of leakage and of electron currents in the triode, the grid potential does not in general remain constant while the boom is moving from one contact to the other, but drifts toward some particular value. A 0.01μf condenser *C* is employed to increase the electro-

static capacity of the grid so that its potential drifts more slowly, and the resistance L of about 10,000 ohms reduces the charging current of the condenser to such a value that sparking does not occur at the galvanometer contacts. A further advantage of using the condenser is that its slow discharge prevents the possibility that chattering at the galvanometer contacts may be passed on to the relay. The arrangement in Fig. 6A has been found the simplest and most generally satisfactory for use with a double contact galvanometer. If the galvanometer has only a single contact, the drift of the grid potential may be taken advantage of by the method of Fig. 6B or of Fig. 6C. Here a resistance[19] of several megohms, L_2 or L_3, takes the place of the missing contact. With the remaining contact open, the potential of the grid drifts toward that of the point to which the high resistance is connected, at a rate depending chiefly on the value of this resistance and of the condenser. Thus, in 6C, the relay closes when the galvanometer contact closes (except for the very short interval required for the condenser to discharge through the low resistance L) and does not open again after the galvanometer contact has opened until the condenser has had time to charge through the high resistance L_3. The lag in seconds on opening, neglecting leakage, etc., will be of the same order of magnitude as the product of L_3 in megohms into C in microfarads. That is, if L_3 is 10 megohms and C 0.1μf, the relay will open about 1 second after the galvanometer contact opens. A lag of about one second or less has been found generally satisfactory. The action in the case of Fig. 6B is similar except that the relay opens when the galvanometer contact closes, and closes again of its own accord after a predetermined time. Since in these two cases a larger capacity is used at C than was used in Fig. 6A, a higher resistance, 50,000 to 100,000 ohms, is desirable for L because of the greater quantity of electricity required to charge the condenser.

Fig. 6D shows another arrangement for double contact galvanometers that may prove useful where the insulation of the boom or grid is low or where the galvanometer contacts are made at infrequent intervals. Here the grid potential drifts toward some point intermediate between the 0- and -12-volt points, depending on the resistances of L_2 and L_3. This value is so chosen that the plate current is sufficient to keep the relay closed if it is already closed but not sufficient to close it if it is open. Thus the relay opens and closes only when the galvanometer makes the appropriate contact regardless of the length of time between

[19] One of the "grid leaks" used in radio receiving sets.

contacts. In this arrangement we have used resistances of 4 and 5 megohms respectively for L_2 and L_3 with a condenser of 0.01μf.

Figs. 6E and 6F show additional arrangements for the single contact galvanometer. The former was described by Beaver and Beaver[11] as an intermediate relay for use with a mercury thermostat; it was their paper that suggested to White of the Geophysical Laboratory the use of a triode in our thermostat. Fig. 6F is our modification of their scheme as applied to White's thermostat. The change was made to prevent the furnace from overheating in case the galvanometer should fail to make contact. This feature is shared by the arrangement of Fig. 6B.

Any or all of the resistances shown in Fig. 6 may be of the type used as "grid leaks" in radio receivers. These leaks are manufactured with resistances as low as 0.1 megohm. For the lower resistance L the ink-soaked paper resistor in one of them may be replaced by a narrow strip of drawing paper heavily blackened with a soft lead pencil. Compact mica or paper condensers are made for radio receiving purposes in capacities up to 0.01μf, and somewhat larger paper condensers in capacities of 0.1μf or over.

In the arrangement of Fig. 2 the electrical energy that operates the triode and relays is obtained from the 110 volt direct current line. The resistances in the filament circuit are so chosen that the potential of the point e is about 12 volts negative with respect to the negative end of the filament; this leaves about 98 volts effective in the plate circuit, while the full 110 volts is available to operate relay R_2. The current through the filament is variable by means of a 200 ohm rheostat (a radio "potentiometer") connected in parallel with it.

Another arrangement, where a direct current line is not used, is shown in Fig. 7. The galvanometer G, triode T, relays R_1 and R_2 and fixed resistance R are the same as in Fig. 2 except that R_2 is wound with coarser wire, to operate on 12 instead of 110 volts. The filament of the triode is heated by current from the 6-volt secondary of a small bell ringing transformer BT[12] and its temperature adjusted by means of a 10 ohm rheostat in series with it. The galvanometer-grid connections are the same as in Fig. 6A except that the grid return is connected to the junction of two 100 ohm resistances r and r' connected in series

[11] Beaver and Beaver, J. Ind. Eng. Chem., *15*, p. 359; 1923.

[12] The smallest size Thordarson "Three Way" bell ringing transformer has been found satisfactory, although it runs rather warm. A "Wayne" bell ringing transformer of about the same size, having a 12 volt secondary, burned out after a few hours' run, indicating that a 6 volt secondary would probably have been satisfactory.

across the filament terminals. The junction of these two resistances is always at the mean instantaneous potential of the filament; as a consequence the mean of the potentials between the grid and the various portions of the filament is not influenced by the alternations of emf at the filament terminals. In order that the grid potential may always be negative with respect to every part of the filament, a two cell (3 volt) flash light battery B_3 is connected between the junction of r and r' and the grid return, the latter terminal being negative. The current through the relay R_1 and the plate is taken from the 110 volt alternating current line, the triode itself acting as a rectifier; the 1μf condenser C_2 connected in parallel with the relay increases the current through the coils of the latter and prevents its being noisy. Relay R_2 is operated by current from the 12 volt[13] biasing battery and should therefore be wound to a lower resistance than was the corresponding relay in Fig. 2: 250 ohms is

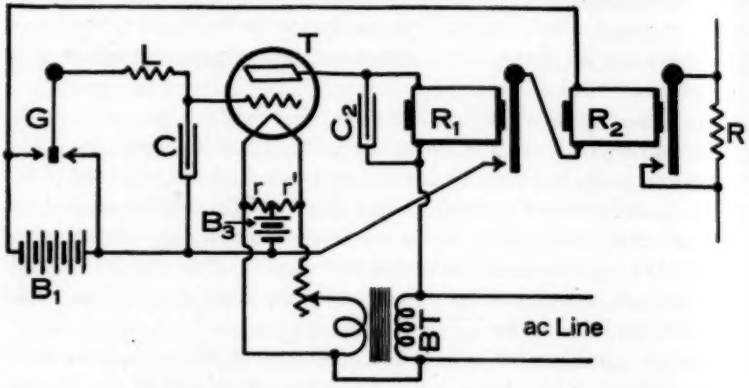

FIG. 7.—*A method for operating the triode from an alternating current line.*

satisfactory for a telegraph relay. A set of 10 standard six inch dry cells may be expected to give at least 1000 hours of continuous service[14] as a biasing battery, assuming that current is flowing through R_2 half the time.

If a 220 volt direct or alternating current line is available for the plate current supply, relay R_2 of Fig. 2 or Fig. 6 may be omitted and the lamp bank R connected directly to the contact terminals of R_1. This saves one relay and makes it possible to use a radio "B" battery in place of the more expensive dry cells of Fig. 7. Under these conditions relay R_1

[13] 20 to 30 volts will be required if 220 volts is supplied to the plate circuit.

[14] See Bur. Standards Circ. 79, "Electrical characteristics and testing of dry cells."

under the form described in the next paragraph, should give satisfactory service if the current supplied to the bridge is 6 amperes or less. With a plate voltage of 110 the current through the contacts of R_1 should not be allowed to exceed about 2 amperes.

The relays R_1 and R_2 of Fig. 2 and Fig. 7 are "main line" telegraph relays.[15] The magnets of R_1 are wound to a resistance of about 20,000 ohms with No. 40 B & S gage enameled wire. R_1 operates satisfactorily with a current of 0.5 milliampere through the coils and R_2 may be connected directly to a 110 volt direct current line, giving a firm pressure on the contacts. In both relays the original contacts were replaced by molybdenum buttons having flat contact surfaces about 2 mm square. These are found to give longer service without attention than the original silver or alloy contacts. Tungsten will carry less current without overheating because, being harder, the actual area of contact is less. With these molybdenum contacts and when tungsten lamps are used as described above for the extra resistance R, relay R_2 may be expected to handle currents up to 15 amperes without too rapid wearing of the contacts. For heavier currents or where the relay is to be actuated by alternating current one of the power relays or remote control switches on the market will probably be better.

CONCLUSION

We have thus far considered only the case where the temperature coil and the heater are identical. It may be desirable to separate them in some cases, particularly where the heating element has too low a temperature coefficient or where the furnace is heated by oil or gas. Under these conditions the temperature coil functions only as a resistance thermometer and it will be unnecessary, in fact undesirable, to vary the current through the bridge by means of the relay.

Since the electrical power supplied to the furnace varies as the square of the voltage or as the square of the current, the thermostat may be arranged with an auxiliary furnace to maintain a constant root-mean-square voltage or current for the main furnace. After equilibrium has been reached between the temperature of the auxiliary furnace and that of its surroundings, the mean power supplied the auxiliary during each cycle depends only on the temperature of the furnace and on that of its surroundings. The influence of the surrounding temperature may be made negligible by operating the auxiliary at a high temperature, say 1200°, while the mean resistance of the heater may be kept constant

[15] Obtained from J. H. Bunnell and Company, New York City.

by making the heater of a material, such as platinum, which does not deteriorate at this temperature. For constant voltage the load should be connected in parallel with the bridge, that is, to w and z in Fig. 2; and for constant current, in series with the bridge. This method is useful to eliminate the effect of variable line voltage where the usual arrangement of the thermostat is ineffectual either because the heater has too low a temperature coefficient, as is the case with carbon, or where it is being operated at such a high temperature that it deteriorates rapidly.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
FEBRUARY, 1925.

A Simple Type of Magnetometer.—The ordinary type of magnetometer with fiber suspension is a troublesome instrument to handle because it has to be carefully levelled, there is commonly considerable drift, a static torque due to the suspension is always present, and it is far from being dead-beat. In an endeavor to eliminate these bad features and at the same time to retain the sensitiveness of the magnetometer, the form described below was made after suggestions given by Dr. H. M. Reese and has been found to work satisfactorily in our general physics laboratory.

A cylindrical cork, about 2 cm in diameter and 6 cm in length, was mounted on the stem of a bell-magnet, taken from an ordinary magnetometer, and the whole was given a thin coating of paraffin wax. Obviously, any suitable magnet could be used. On top of the cork was set a plane galvanometer mirror. A test tube, about 3 cm in diameter, was set in a wooden support, the tube filled with water and the magnet with its cork float inserted. The water level was adjusted so as to make the surface film convex upward. When this was the case, we found the magnet would float in the very center of the tube and remain there persistently, while, if the film was concave, the float would be drawn to one side of the tube and held there. Large clearance between cork and tube gave better centering than small clearance, contrary to expectations, and it also made careful levelling of the support unnecessary. A suitable box, with observation window in the side, was set over the tube to cut off air currents. The deflections were read with telescope and scale in the usual way. We found it possible to obtain deflections readable with accuracy when a very weak magnet was set 60 to 90 cm away, and we could detect effects at greater distances. There was practically no preliminary drift, and very small delay in the coming to rest of the moving system at either the zero or deflected position.

H. E. HAMMOND, University of Missouri, Columbia, Mo.

MICROSCOPE ILLUMINATION BY MEANS OF QUARTZ ROD

By S. B. Leiter

There has been considerable discussion of late in regard to the light carrying properties of quartz. Much stress has been laid on the ability of the quartz rod to bend light rays around corners. This in itself would not necessarily set quartz apart as peculiar. Light passing through any transparent rod or tube with reflecting walls will "bend" around a corner, but the absorbing power of quartz is low, the melting point high. These latter characteristics make it a very convenient substance to use for microscopic illumination as will be shown by the following photographs, especially in the case of the inverted type of microscope where illuminating a transparent specimen has presented some difficulties.

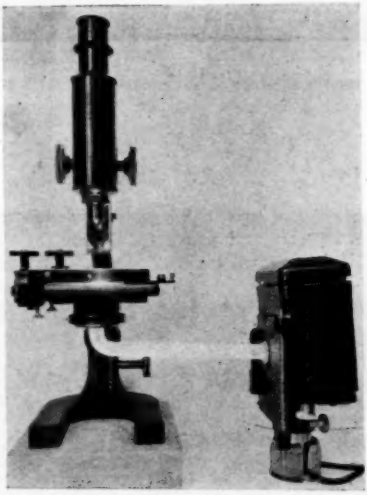

Fig. 1

Fig. 1 shows the arrangement for substage illumination for work with an ordinary small microscope. The lamp at the right is a small moving picture outfit lamp from which the lens has been removed. By means of a half inch quartz rod the light is directed to the stage of the microscope, is always where it is wanted, and there is less loss of intensity than when the light is reflected from a substage mirror.

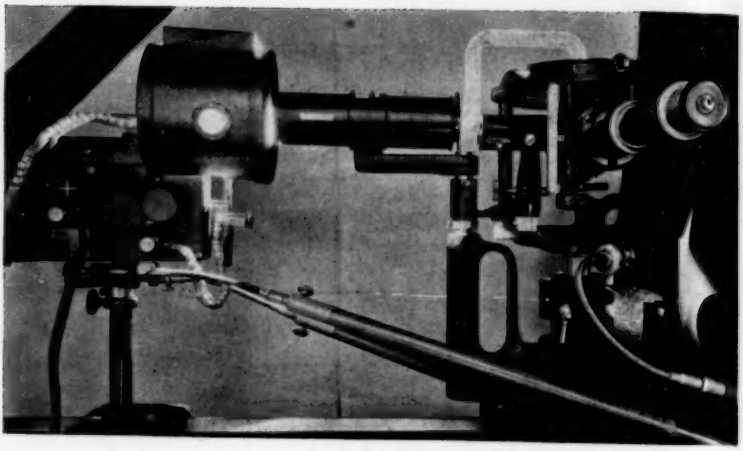

FIG. 2

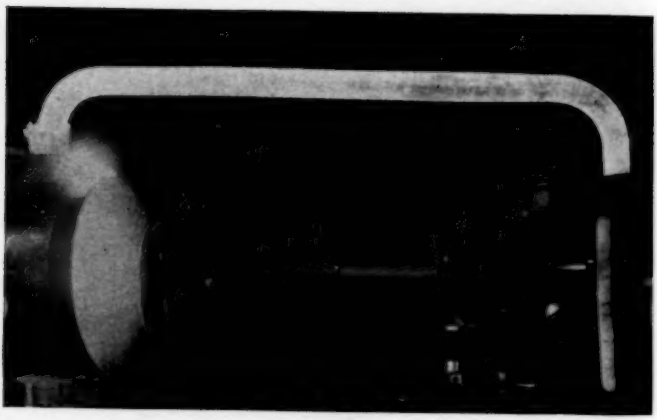

FIG. 3

Figs. 2 and 3 illustrate two methods of carrying the light from an arc to the stage of the inverted type of large microscope. Slightly greater intensity is obtained with arrangement shown in Fig. 3 and in this case the quartz rod can be supported so that with a very slight adjustment it can be brought into use at any time. On the other hand, it has the disadvantage of making necessary some device for preventing the covering over of the end of the rod by the little white ash from the burning of the carbon. However, a small current of air through the top of the casing above the arc should keep the rod clean. The end of the rod toward the microscope should be ground uniformly, but not polished. Although diffused light is thus obtained, it is sufficiently intense for photographic purposes and gives better detail than ordinary methods of illumination.

In this connection I might mention another use for the quartz rod. In any case where the source of light is an incandescent lamp, if the light is allowed to pass through a short length of quartz rod with ground end, all trouble from images of the filament is obviated without loss of light. It is possible to work farther from the source of light in this manner also if one wishes to avoid heating of specimen.

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Susceptibilities of the rare earths.—Complete set of measurements of the susceptibilities of the rare-earth elements from number 57 to number 72, made upon compounds of these elements in which they appear as trivalent. The data are partly from the author's own work and partly from Cabrera's; the agreement, where both have made measurements upon the same element, is generally within 5 per cent, although Meyer expatiates upon the defects of his own method (in particular, the possible magnetization of the other atoms in the compounds is not corrected for). Plotted versus atomic number, the susceptibilities form a curve of striking appearance, with two rounded maxima between elements 59 (Pr) and 60 (Nd) and elements 66 (Dy) and 67 (Ho) respectively, and a deep cusp-like minimum between 62 (Sm) and 63 (Eu). It is evident that these data are very significant for theories of atomic structure. Meyer makes the interesting suggestion that there may be two or more distinct arrangements of electrons for one and the same element, thus accounting for the "elements" which have been defined and described by various investigators, and yet do not coincide in their properties with any of the known elements which fill out all but one of the available places.—[S. Meyer; Phys. ZS. *26*, pp. 51-56, 1925.]

KARL K. DARROW

Absorption of sound in smoke-filled air.—A wire cage 100x40 x56 cm (or 100x40x29 cm) walled with cotton 3.5 cm thick was filled with clear air and then with air laden with tobacco-smoke (the density of the smoke subsequently determined by weighing the amount that settled out). The sealed end of a glass tube, clamped in the middle and excited at the far end by friction with a continuously moving belt, was inserted through the wall at one end and served as source of sound, of wave lengths ranging (with different tubes) from 1.5 to 6.5 cm; a device for measuring sound-pressure was incorporated in the opposite wall. Measuring the intensity of the sound in this manner with clear air and with smoke-laden air, and dividing the difference between the two readings by the first reading, the authors obtain the "absorption," which increases very rapidly with decreasing wave length (in about the ratio 1:3, as the wave length sinks from 6.5 to 1.5). Doubling the density of the smoke doubles the absorption at the highest wave length, and produces a smaller percentage change at the lower ones. To take an example, smoke of density $7.3.10^{-6}$g/cc diminishes the received intensity by 39 per cent in the shorter box and 70 per cent in the longer one. The device for producing the intense high-frequency sound is described in detail, and commended.—[W. Altberg and M. Holtzmann (St. Petersburg); Phys. ZS. *26*, pp. 147-153, 1925.]

KARL K. DARROW

Ionization by alpha-particles in monatomic and diatomic gases.—The total ionization produced in each of the inert gases, in air, in hydrogen, and in nitrogen by alpha-particles, which are admitted to the gas with a definite initial speed (or, in the usual phrase, with a definite residual range) and allowed to continue through it until they lose their speed altogether, was measured by a conventional method for various values of the residual range from 3 to 20 mm (except for Kr and Xe, for which the residual range could be increased only to 7 mm because of scarcity of these gases). In other words, the quantity measured is the number of ions produced, by a given number of alpha-particles, as their kinetic energy is reduced from a known adjusted initial amount to an imperceptibly small amount by encounters with atoms of the kinds specified. For the noble gases considered by themselves, this number increases steadily and almost linearly with the atomic number of the atom; but the diatomic gases fall out of line, the values for nitrogen and air being less even than that for helium. Assuming from prior work that 33 volts (on the average) is spent per ion produced in air, and that the corresponding values for the other gases are inversely as the numbers of ions produced in them, Gurney calculates values ranging from 19.6 for Xe to 33 for nitrogen. The only value among these agreeing closely with the ionizing potential for the valence electron of the atom is that for helium—26.2 volts expended per ion produced, vs. 24.5 for the ionizing potential of the atom.—[R. W. Gurney, Cavendish; Proc. Roy. Soc. *A107*, pp. 332-340, 1925.]

KARL K. DARROW

THE FORMATION OF ALPHA-RAY TRACKS BY SIMPLE MEANS

By Chas. T. Knipp and N. E. Sowers

Alpha-ray tracks for lecture table demonstrations, and also for quantitative study, may be formed by means of exceedingly simple apparatus. A glass containing vessel blown into form from a pyrex crystallizing dish, a tiny source of alpha-rays mounted in a protected cavity at the inner end of an aluminum plug, a battery syringe bulb, a 100 to 200 volt B-battery, and a lantern for general illumination, constitute the essential parts.

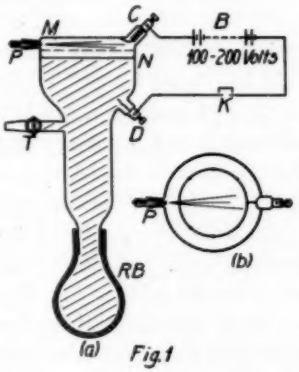

Fig. 1

This simplified apparatus is built along the lines of a modified cloud apparatus described by one of the authors a number of years ago.[1] It is entirely of glass—pyrex glass. Fig. 1*a* shows a vertical section through the axis of the expansion chamber and bulb *RB*, while Fig. 1*b* is a view from the top downward. Several ray tracks are indicated as issuing from the tip of the plug *P*. In Fig. 1*a* also are shown the electrical connections. The main body of the expansion chamber is made of a crystallizing dish since these vessels have square corners and the bottoms are quite plane and free from undulations which might produce troublesome distortions when viewing the ray tracks from above. To the lip of the crystallizing dish is fused a narrower tube, some 15 cm in length, to the lower end of which is attached a rubber syringe bulb, *RB*, as shown in Fig. 1*a*.

[1] Science, *3*, p. 930; Dec. 24, 1909.

The expansion chamber, MN, constitutes the upper end of the apparatus. Two binding posts, C and D, making electrical connections to the electrostatic field plates M and N respectively are shown. The upper plate is the inside bottom of the inverted crystallizing dish, and electrical connection is made by a loop of fine platinum wire laid flat on the glass and fused to it at 6 or 8 points, as shown in Fig. 1*b*. The lower plate is simply the surface of the water which is slightly acidulated to render it a better conductor. A tap, T, for admitting water into the vessel is attached. The alpha-rays emerge through a thin aluminum window at the inner end of the aluminum plug, P. By making the walls of the plug heavy the rays are confined to a beam proceeding directly across the expansion chamber as shown in Fig. 1*b*. The plug P is ground into the glass seat and held in position by stop cock grease.

The necessary electrical connections are shown in Fig. 1*a*. For best results 200 volts should be placed in the circuit, though good results are obtained by connecting directly to a 110 D.C. source. In making the electrical connections it is not necessary to pay any attention to polarity or to earthing one pole of the battery, nor is it necessary, as commonly supposed, to arrange for *closing* or *opening this circuit* at proper intervals when producing the alpha-ray tracks. Hence a shorting key K is included in the circuit for closing the electric circuit permanently while operating the apparatus. The electrostatic field between M and N sweeps away continuously the ions formed by impact of the alpha-particles, while the ions formed just at the instant the expansion is right, acting as nuclei for condensation of water vapor, grow rapidly and become visible. Because of their increased mass they are but little, if any, affected by the electrostatic field, remain stationary for a brief instant and then settle down under the action of gravity. For purposes of illumination and making the tracks visible a flat beam of light from a lantern should pass horizontally between the plates M and N.

The procedure in operating the apparatus is briefly as follows: Fill the bulb and containing vessel with water up to the level N. Insert the plub P, and connect the source of D. C. potential to the binding posts C and D, including a key K. Before closing the key the vessel should be tipped so as to wet the surface M, thus forming the upper electrostatic field plate as a film of moisture, which, since the water was made slightly conducting, makes connection to the binding post C by the platinum wire loop previously referred to. With the illumination on and the key K closed compress the bulb RB and release it alternately

until the proper expansion ratio is obtained, whereupon the tracks will appear freely, being very distinct and persisting for some moments. The expansion chamber MN being of large diameter and small height (the ratio being approximately 8:1), and free from projections or curved surfaces within, insures minimum turbulency. When the expansion is just right (and this is readily gotten by trial) the tracks appear exceedingly clear cut, oftentimes extending entirely across the expansion chamber. The tracks may be projected upon a screen.

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URBANA, ILLINOIS,
NOVEMBER, 1924.

Velocities of Photoelectrons from Platinum Films.—This is a contribution to the old question as to whether photoelectrons escaping from a metal surface facing the incident light, which therefore move initially contrary to the light, have a different energy-distribution from photoelectrons escaping from a metal surface which the light reaches by coming up through the metal from behind, so that they initially move in the same sense as the light. Chien Cha, working on platinum films deposited on quartz and illuminated by ultraviolet light, and measuring the potential which the insulated film assumes when illuminated, finds no difference in the maximum speed of the escaping electrons in the two cases, if the film is formed by sublimation. But if the film is formed by cathode-sputtering, there is a serious difference, the maximum kinetic energy of the electrons being nearly 20 per cent greater when the light reaches the uncovered metal surface from behind than when it falls upon the surface from before. This difference diminishes as the film thickens. It is significant that the maximum kinetic energy of the electrons from sublimated films, plotted versus frequency, conforms to the well known linear relation with h for its slope; while the corresponding relation for the electrons from sputtered films is not even linear. This is to be set down as an important difference in behavior between sputtered and sublimated films. [Chien Cha, Chicago; Phil. Mag. *49*, pp. 262-272, 1925.] KARL K. DARROW

Absorption of quanta in the atom where they originate.—In Auger's experiments by the C. T. R. Wilson method upon argon and krypton irradiated by x-rays, he observed numerous instances of apparently double and even triple ionization of the same atom, which he attributes to this process; the primary quantum expels a K-electron from the atom, an L-electron descends to take its place, causing the emission of a quantum which detaches an M-electron, and there may be an additional similar stage. Thus in experiments upon argon atoms floating in a great excess of hydrogen, he sometimes observed two electron paths emanating from the same point, the longer being presumably

that of the *K*-electron and the shorter that of the *L*-electron; while when he worked with krypton atoms mingled with an excess of hydrogen, he sometimes observed three electron-paths from the same point, the shortest being 1 mm long when the medium one was 15 mm long. If the primary quanta are not of too high frequency, the *K*-electron and the *L*-electron may have paths of about the same length. De Broglie and Thibaud, whose method consists essentially in the deflection of these same expelled electrons by a magnetic field and reception of them by a photographic plate, also find that electrons produced by this process exist (not a new result) and are more common (in comparison with the ones ejected directly by the primary quantum) than might be expected; and the abundance of these increases with the atomic number of the irradiated atom. Further, an electron is more liable to be ejected by a quantum issuing from the nucleus of its own atom, than by quanta coming from the nuclei of other atoms. [P. Auger, C. R. *180*, pp. 65-68, 1925. M. de Broglie and J. Thibaud, C. R. *180*, pp. 179-180, 1925].

KARL K. DARROW

Trial of the classical dispersion-formula by the data for the noble gases.—After a comparison and estimate of the various sets of data establishing dispersion-curves for He and A in the visible and near ultraviolet, and citation of the solitary sets of data for Ne, Kr, and Xe in the visible, Herzfeld and Wolf test the suitability of the classical dispersion formula to express these data, ascertaining in each case the best possible values for the disposable constants of that formula. Assuming a single kind of oscillator with a single frequency, they obtain perfect agreement (i.e., within the limits of the uncertainties of the data) for all the gases but A, and only a slight discord for this last; but the values which must then be assigned to the natural frequencies of the oscillators are far away in the ultraviolet, beyond even the limit of the principal series. The authors would like to say that each lies at the wave length where the continuous absorption of the gas, beyond the limit of the series, has its maximum; but the data scarcely sustain this. The values for the number of oscillators per atom, determined by the other of the two disposable constants, are 1.1, 2.4, 4.4, 4.9 and 5.6. The authors also tried out a dispersion-formula of the classical type, taking account of two kinds of oscillators with two natural frequencies, whereof they arbitrarily put one equal to the resonance-frequency. Neither for He nor for A was the agreement any better than before and the newly-introduced frequency, far more important in its influence on the curve than the resonance-frequency, also lay beyond the limit of the principal series. In discussing the mercury spectrum the authors point out an apparent case of anomalous dispersion not accompanied by absorption. —[K. F. Herzfeld and F. L. Wolf, Munich; Ann. d. Phys. *76*, pp. 71-105, 1925].

KARL K. DARROW

A MAGNIFYING STEREOSCOPE AND CAMERA LUCIDA; TWO INSTRUMENTS FOR AIRPLANE MAPPING*

By I. C. Gardner

The two instruments to be described below were designed and constructed for use by the Coast and Geodetic Survey in connection with mapping by airplane photography. In Fig. 1 there is shown a stereoscope having variable magnification. This instrument is not designed

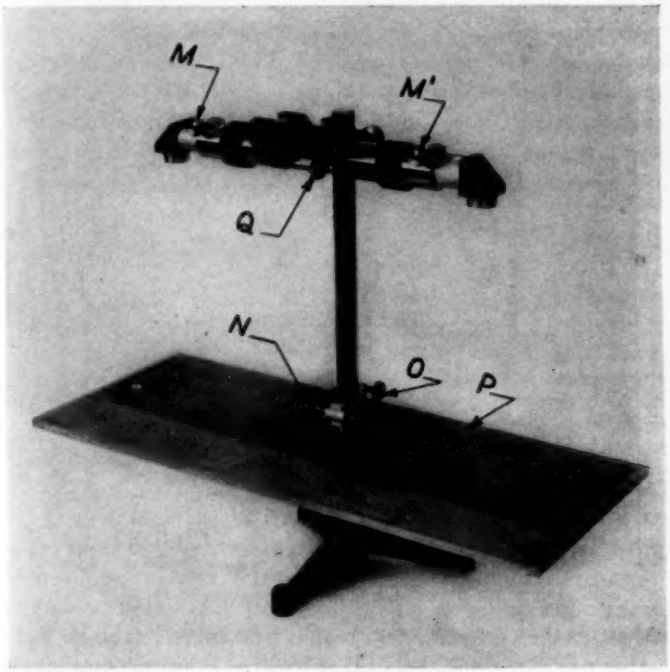

Fig. 1. *Magnifying stereoscope.*

for the making of measurements on a photograph, but for close stereoscopic inspection of a pair of photographs in order that landmarks which have been used in a triangulation may be identified. Flag poles, church steeples, and similar objects are frequently used as stations in a

* Published by permission of the Director of the Bureau of Standards of the Department of Commerce.

survey. During the course of a survey these objects are prominent landmarks but when sought for in the airplane photograph they are very inconspicuous. The stereoscope was designed in order to overcome this difficulty. Two exposures of the same territory taken in succession from an airplane are viewed stereoscopically. The two photographs placed on the platen P and held down by ring shaped brass weights not shown in the illustration are adjusted until the stereoscopic fusion into a single image is obtained. The variation in magnification is accomplished by varying the conjugate distances from objective to photograph and from objective to image plane. The distance from objective to image plane is varied by the racks and pinions actuated by the two wheels M, M'. Focusing is accomplished by elevating or lowering the entire platen by the wheel N and clamp screw O and the interpupillary distance may be adjusted by the knurled head Q, which actuates the two eye-pieces symmetrically by means of a right and left hand screw.

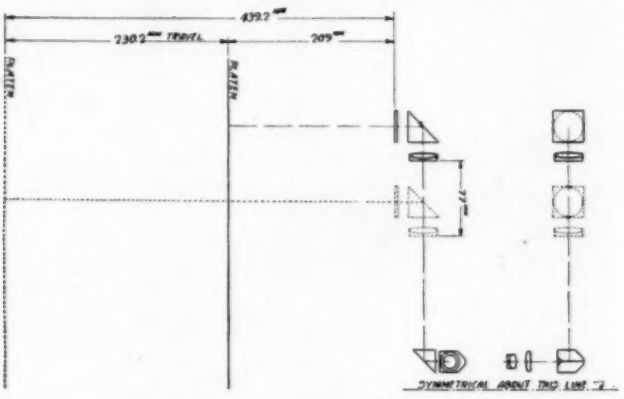

FIG. 2. *Optical systems of magnifying stereoscope.*

One-half of the optical system is shown in Fig. 2. The E. F. L. of the eye piece is 25 mm and that of the objective 125 mm. One roof-prism and two right angle prisms are used to erect the image and bend the optic axis as desired. The range of motion of the platen is 230 mm, which enables the magnification to vary from 3.5 to 9.5 diameters.

The second instrument, shown in Fig. 3, is a camera lucida with adjustments permitting the transfer to a map and rectification of points on an airplane photograph. Two right angle prisms are cemented together to form a cube after one of the hypotenuse surfaces has been half silvered. The prism is mounted in the casting at T and permits

the operator to look through the half silvered surface directly upon the map to which the points are to be transferred. The map is mounted upon the drawing board inclined in a convenient position for use. The platen *U* carries the airplane photograph and its image is projected upon the map as is usual with a camera lucida. The rays from the photograph are reflected by the mirror *V* before passing through the ocular prism in order that it need not be necessary to use a reversed photograph. When the distance from image and photograph to ocular prism

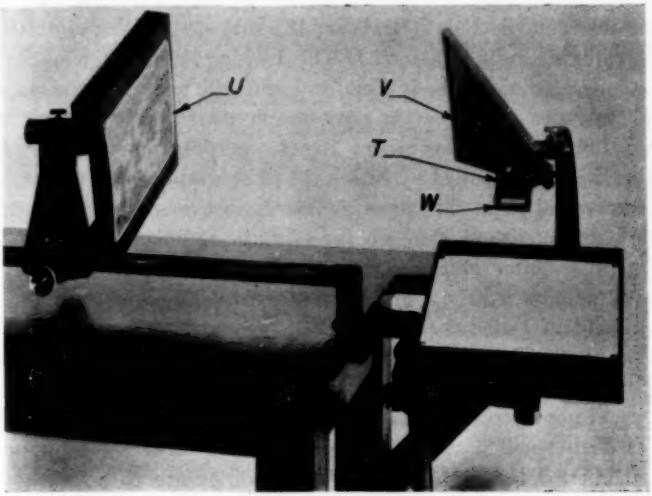

FIG. 3. *Camera Lucida.*

are not the same the image of the picture is not in the plane of the map and there will be parallax between the two. To diminish this a cap is mounted above the ocular prism carrying a small aperture through which one views the map. The center of this aperture is the center of projection, both for the map and for the photograph. Filters are provided which may be inserted in the path of the rays either between the ocular prism and photograph or between the ocular prism and map to properly balance the illumination of the two surfaces for convenient work. One filter is shown in position at *W*.

In use the photograph is mounted centrally on the platen which is adjusted along the horizontal rod until the equivalent air path length from eye aperture to photograph is equal to the equivalent focal length of the photographic objective with which the exposure was made. The

length of the horizontal rod and range of adjustment of the platen are such as to enable this setting to be accomplished for focal lengths lying between 20 and 1500 cm. The platen is mounted upon a horizontal axis by which the photograph may be rotated into any azimuth and is further carried by a universal joint which permits it to be inclined, by which the rectification of the photograph, i.e., the compensation of the distortion introduced by any roll or pitch of the airplane, is accomplished. The rectification is arrived at empirically. The true configuration of at least four points on the photograph must be known. These are then plotted to scale on the map to which points on the photograph are to be transferred. The drawing board carrying the map is then adjusted along the inclined rod until the points on the map and the homologous points on the photograph most nearly coincide when viewed through the ocular prism. Exact superposition of all four points can then be accomplished by tilting the photograph about the two axes of the universal joint. The drawing board must be always kept vertical to the axis normal to the two faces of the prism and which passes through the center of the eye aperture. After the adjustments are correctly made when the four known points on photograph are brought into coincidence with the four plotted points all other points on the photograph may be transferred directly and there will be no distortion other than that arising from difference of elevation of the points and distance of points from center of photograph. Both of these distortions may be kept small if only the central portion of each photograph is used. When details from several photographs are to be transferred to one map, it is found that the scales of the different photographs may vary because of the different heights from which the photographs are taken. This variation in scale is compensated for by racking the drawing board up or down the inclined rod.

The writer desires to acknowledge his obligation to Mr. F. A. Case for the care and skill shown in executing the details of design of these two instruments and to Mr. O. G. Lange and Mr. Frederick Knoop for the construction of the magnifying stereoscope.

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